

Teachers' Professional Learning Within Communities
of Mathematics-for-Teaching Practice

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Abstract

Ontario bansho is an emergent mathematics instructional strategy used by teachers working within communities of practice that has been deemed to have a transformational effect on teachers' professional learning of mathematics. This study sought to answer the following question: How does teachers' implementation of Ontario bansho within their communities of practice inform their professional learning process concerning mathematics-for-teaching? Two other key questions also guided the study: What processes support teachers' professional learning of content-for-teaching? What conditions support teachers' professional learning of content-for-teaching? The study followed an interpretive phenomenological approach to collect data using a purposive sampling of teachers as participants. The researcher conducted interviews and followed an interpretive approach to data analysis to investigate how teachers construct meaning and create interpretations through their social interactions. The study developed a model of professional learning made up of 3 processes, *informing with resources*, *engaging with students*, and *visualizing and schematizing* in which the participants engaged and 2 conditions, *ownership* and *community* that supported the 3 processes. The 3 processes occur in ways that are complex, recursive, nonpredictable, and contextual. This model provides a framework for facilitators and leaders to plan for effective, content-relevant professional learning by placing teachers, students, and their learning at the heart of professional learning.

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CHAPTER ONE: INTRODUCTION TO THE STUDY

This study investigated the ways that individual teachers who were members of communities of practice navigated the phenomenon of teaching using Ontario bansho and thereby added to their knowledge construction of mathematics-for-teaching. This knowledge construction is critical as teachers move from the paradigm that saw mathematics instruction as the transmission of efficient algorithms from teachers to students toward the constructivist paradigm that values the mathematical sense-making students naturally do and seeks to build on that sense-making activity in intentional ways. Many existing efforts to build this teacher knowledge have themselves been built on efforts to inform teachers in professional learning situations that resemble traditional classroom methods of instruction in which a leader enumerates key learnings. This study examines and makes visible the ways that teachers added to their knowledge of mathematics-for-teaching within communities of practice.

Background

The traditional focus of teaching mathematics has been on the transmission of mathematical content. In elementary schools, this approach primarily consists of the teaching of standard methods (algorithms) for obtaining correct answers to questions involving the addition, subtraction, multiplication, and division of whole numbers, fractions and decimals, measurement formulae geometric rules and the like. Fosnot (2007a) likens this approach to the teaching of a somewhat obsolete discipline like Latin, in that it is divorced from the real world.

Beginning in the 1950s, educators were concerned that students graduating from high school lacked the necessary proficiency to do basic math. There was also fear of a growing deficit in the number of capable mathematicians and scientists (Klein, 2003). In North America, the National Council of Teachers of Mathematics (NCTM) undertook the massive task of studying the mathematics education research and in 1989 launched their *Curriculum and Evaluation Standards*, outlining the mathematical understanding, knowledge, and skills that students should acquire from Pre-K through grade 12. This document put constructivism on solid ground in the mathematics education community, and by the end of the 20th century virtually every jurisdiction in North America adopted a constructivist approach for mathematics teaching and learning (Small, 2008).

Small (2008) states that “the two goals of a constructivist approach to mathematics are students’ opportunity to develop richer and deeper cognitive structures related to mathematical ideas, and students’ development of a level of mathematical autonomy” (p. 4). In a constructivist classroom, interactions with other students and the teacher enable students to construct new mathematical knowledge as they are given the opportunity to articulate their own thoughts.

Small (2008, pp. 5-7) points out that although adoption of a constructivist approach is presently widespread, there continues to be discussion about what mathematics itself is. Some continue to see mathematics primarily as a set of procedures. Other people see mathematics as a hierarchy of concepts and skills, although they agree that there is no single definitive sequence for the teaching of these concepts and skills. Others suggest that mathematics is a study of patterns that allow us to interpret many situations. Still others consider mathematics to be a way of thinking that

involves mathematical process thinking skills that are cultivated as students develop their abilities to think mathematically about problems. Understanding and honouring these multiple perspectives also contributes to the construction of teachers' specialized knowledge of mathematics teaching.

Although an intended curriculum may reflect a constructivist approach, what is enacted in individual classrooms by individual teachers often reflects the effects of traditional approaches to mathematics instruction. Stodolsky's (1988) study of 39 5th-grade classrooms in the 1980s showed that

math instruction places all but the exceptional student in a position of almost total dependence on the teacher for progress through a course. In essence, the traditional math classes contain only one route to learning; teacher presentation of concepts followed by independent practice. (As cited in Ellis, 2008, p. 1338)

Hiebert (2008) showed that although U.S. teachers used rich problems in their instruction, they taught them in didactically traditional ways, telling students how to solve the problems using the algorithms the teachers gave them. He states that this finding reflects the need for teachers to re-schematize their thinking about the benefits to mathematical sense-making that the solving of problems affords the learner.

This re-schematization is a goal of mathematics professional learning. Ball, Hill, and Bass (2005) assert that

most teachers are graduates of the very system that we seek to improve. Their own opportunities to learn mathematics have been uneven, and often inadequate.

Studies over the past 15 years consistently reveal that the mathematical knowledge of many teachers is dismayingly thin. We are simply failing to reach reasonable

standards of mathematical proficiency with most of our students, and those students become the next generation of adults, some of them teachers. (p. 14)

However, Small (2008) claims that “as teachers become more familiar with which ideas are more complex for students and why, they are better able to ensure that their instruction is at the appropriate developmental level for students” (p. 12).

This complex learning on the part of teachers is referred to as the knowledge of mathematics-for-teaching. Ball et al. (2005) state that

knowing mathematics for teaching demands a kind of depth and detail that goes well beyond what is needed to carry out the algorithm reliably. ...There are predictable and recurrent tasks that teachers face that are deeply entwined with mathematics and mathematical reasoning—figuring out where a student has gone wrong (error analysis), explaining the basis for an algorithm that children can understand and showing why it works (principled knowledge of algorithms and mathematical reasoning), and using mathematical representations. Important to note is that each of these common tasks of teaching involves *mathematical* reasoning as much as it does pedagogical thinking. (p. 21)

Local jurisdictions continue to focus their efforts on effecting this change in knowledge of mathematics-for-teaching.

Problem Context

Ball et al. (2005) claim that mathematics teachers today have not themselves constructed the necessary depth and detail of mathematical understandings to build a complex knowledge of mathematics-for-teaching. Finding effective ways to reveal the need for teachers to consider and re-form their schema about what constitutes effective

mathematics instruction has been the goal of mathematics professional learning in Ontario. Ellis (2008) states that traditional modes of professional learning consisted mostly of mathematics consultants and teachers telling other teachers the best way to tell students how to carry out standard algorithmic procedures (p. 1338). Following a review of the professional learning literature, the Ontario Ministry of Education's *Early Math Strategy: The Report of the Expert Panel on Early Math in Ontario, 2003* recommended changes in the approach of professional learning, stating

professional development should involve teachers in working on mathematics content and focusing on key concepts, exploring ways of determining a student's prior knowledge, and finding ways of providing connections to that knowledge in future learning. As well, teachers need to develop their understanding and experience of effective teaching strategies in mathematics. ...Professional development should include the opportunity to link their new experiences with work in their own classrooms. This means trying out new strategies with their students and then having the opportunity to connect with colleagues to share their stories and to seek ways to continue their growth. (2003, p. 49)

Since 2003, school boards in Ontario have been working to implement these recommendations, and their work has been supported with funding, resources, and professional learning support directly to boards and teachers through the Literacy and Numeracy Secretariat (LNS). Boards have implemented some large-scale professional development and in the last 3 years have focused attention on the development of small-scale professional learning groups. The intent of these groups has been to develop

knowledge of mathematics-for-teaching among interested teachers through collaborative practices that link practice with theory.

These groups have engaged in various forms of collaborative work among teachers who identified themselves as willing to make changes in their instructional practices in mathematics. Groups of teachers of mathematics at the elementary level have engaged in activities ranging from book studies to classroom-based lesson study and co-teaching, all with the goal of developing increased knowledge of mathematics-for-teaching among teachers and facilitators of teachers' professional learning in mathematics.

From this collaborative work, interest emerged in the ways other countries approach mathematics instruction. The work of Stigler and Hiebert (1999) inspired some groups to study the way the Japanese approach the teaching of mathematics. In Japan, teachers gather routinely to collaboratively design lessons, observe the lessons as they are taught to a class, collaboratively make revisions to the lesson, teach the lesson again, and then make public the reports of this action research in well-archived repositories. Takahashi (2006) states that this research is done once or twice per year by several groups of teachers in each school. The archives are massive and provide rich research upon which curriculum writers may draw.

One key feature of Japanese lesson planning is how teachers approach the use of blackboard or whiteboard as a central part of their mathematics instruction. This writing is called *bansho* (bansho means board writing). Kubota-Zarivnij (2007) states that bansho is used to show the flow of the lesson process and to connect parts of the lesson coherently together in order to:

- build student understanding;
- keep a record of the lesson;
- help students remember what they need to do and think;
- help students see connections of different parts of the lesson and the lesson progression;
- contrast and discuss ideas students presented; and
- organize student thinking and develop new ideas. (p. 3)

Ontario has adapted the idea of the public record of the class's collective thinking to fit our culture. Our mathematics community values a constructivist approach to mathematics instruction. This means that teachers should value the emerging strategies students use to solve mathematical problems, realizing that they fit along a continuum or sequence of increasingly sophisticated solution strategies. This progression of strategies becomes the public record, or the bansho, of the class's collective thinking. A final statement serves as the generalization of the concept of the lesson and is an important conceptual idea upon which further learning can be built in subsequent problem-based lessons.

The problem facing facilitators of teachers' professional learning of mathematics has been to find effective ways to bring together research and practice. Their queries centre around how to draw together processes and conditions that shift paradigms, advance the learning of mathematics-for-teaching, support research-informed instructional practices, and ultimately effect increased student achievement.

Purpose and Empirical Questions of the Study

It is important to know how to build teachers' knowledge of mathematics-for-teaching so that all students will have access to significant mathematics learning. Knowing the conditions and processes that have contributed to teachers' construction of this knowledge is important so that designers of professional learning can facilitate the professional learning paradigm shift needed in mathematics education. Kubota-Zarivnij (2007), representing the Literacy and Numeracy Secretariat (LNS) of Ontario, makes the claim that Ontario bansho is "a professional learning strategy that develops teachers' knowledge of mathematics *for teaching*" (p. 21). The purpose of this study is to examine the experiences of teachers who have adopted this approach in their communities of practice and to determine the conditions and processes that enable this strategy to build the teachers' knowledge of mathematics-for-teaching. This study addresses Kubota-Zarivnij's claim through the question, "How does teachers' implementation of Ontario bansho within their communities of practice inform their professional learning process concerning mathematics-for-teaching?" The following two key questions framed the study: What processes support teachers' professional learning of content-for-teaching? What conditions support teachers' professional learning of content-for-teaching?

Rationale

Reeves (2000) says that the single largest contributing factor to student achievement is the quality of teaching. Mathematics education researchers such as Van de Walle (2001), Fosnot and Dolk (2002), Fosnot (2007b), Hiebert (2008), Boaler and Humphreys (2005), Small (2008), and many others agree and state further that there is a

body of knowledge which mathematics educators need that is different from either mathematicians' knowledge of mathematics or the knowledge the general public has of mathematics. Ball (as cited in Literacy and Numeracy Secretariat [LNS], 2005) refers to this as specialized professional knowledge that is not known by people who are otherwise well educated. She maintains that it is a kind of unnatural knowledge, in that mathematics teachers are concerned primarily with learning to discern the way that others (i.e., primarily their students, but also parents and colleagues) think and reason about mathematics. Professional teacher education, she suggests, equips teachers to know what they need to know in order to be able to do the specialized work of teaching mathematics (LNS, 2005, video file time code 34:30).

Hiebert (2008) suggests that this learning of mathematics-for-teaching is an ongoing process that teachers of mathematics engage in daily to some degree: "Every day, thousands of teachers spend hours teaching and they learn, at least a little, about how to improve. But the learning is haphazard and not shared with colleagues" (p. 15). The Ontario Ministry of Education, district school boards, and Ontario's two provincial subject associations contribute to a growing network of mathematics educators by engaging groups of teachers who work together to learn more about instructional strategies that have the potential for yielding increased student understanding in mathematics. The LNS and many district school boards support teacher learning in classroom-embedded collaborative professional learning situations, suggesting that teachers learn research-informed teaching strategies (including Ontario bansho) in collaborative learning groups. As they implement the strategies in their classrooms and meet again several times in their collaborative groups, they discuss, reflect upon, and

consolidate their ongoing learning. These learnings are then shared provincially through the activities of the Ministry of Education and provincial subject associations. Through these efforts it is expected that the collective knowledge of mathematics-for-teaching will increase among teachers of mathematics; it is also expected that the result will be increased student achievement in mathematics. This study will be able to inform the ongoing work of the Ministry of Education and district school boards as they continue to work with groups of teachers and administrators to create a provincial culture of collaborative inquiry.

In Ontario, an increasing number of teachers are interested in implementing Ontario bansho. In addition, as of August 2010, Ontario bansho is being proffered as a research-informed instructional strategy and is being recommended to teachers and administrators as a new instructional strategy to be learned and used as part of the School Effectiveness Framework (SEF), which is mandated for use in the reporting of boards to the Ministry of Education for accountability purposes. There is no research into the effectiveness of Ontario bansho specifically in terms of how implementing this strategy contributes to teachers' professional learning of mathematics-for-teaching. This study would be an initial investigation into its efficacy in terms of teacher learning.

Of note to facilitators of professional learning within boards, this study has potential to confirm the claims of the Literacy and Numeracy Secretariat that Ontario bansho is an effective learning strategy not only for students but also for teachers. In other words, Ontario bansho is “good bang for the buck,” allowing teachers insight into the development of their own knowledge of mathematics-for-teaching. By extension

then, the facilitators themselves will add to their own knowledge of mathematics-for-teaching, insofar as they participate in Ontario bansho with teachers and students.

There is also potential for this study to have an impact on the international mathematics community. There are numerous opportunities to share research at international conferences; I am a member of both the National Council of Teachers of Mathematics (NCTM) and the National Council for Supervisors of Mathematics (NCSM) and presented my study on Ontario bansho at the NCTM international conference in San Diego in April 2010. There is also potential to impact the international community of mathematics educators through the submission of articles for publication in the journals of both of these organizations.

Conceptual Framework

The conceptual organization that frames the study rests on a theory of constructivism that holds that both teachers and students construct their understanding of mathematics teaching and learning. Constructing understanding of the complex components of knowing mathematics-for-teaching involves considering conditions and processes that contribute to the ways in which the participants in the study learned and developed meaning as they implemented Ontario bansho in their communities of practice. The conditions support learning in complex ways that contribute to the participants' feelings of safety, motivation, and satisfaction and act as support to persevere when the rigor of learning new strategies and constructing new ideas becomes challenging. The processes are actions that contribute to the development of meaning as new ideas are integrated into the participants' thinking about what it means to them to be effective mathematics teachers.

Outline of Remainder of the Document

This report is composed of five chapters. Chapter 1 has served to introduce and to give background for the study as well as to outline the study's purpose, conceptual framework, and key questions. Chapter 2 reviews related literature. This chapter is organized by the conceptual framework and discusses the themes, processes, and conditions that exist within that framework. In Chapter 3 I describe the methodology for the study, give rationale for conducting the study in this manner, and outline the analysis of the data that were collected in the study. I present the results of the investigation in chapter 4; these results are framed by conceptualizing the processes and conditions that emerged from the analysis of the data. Chapter 5 contains a discussion of the results and conclusions I have drawn from the results. In this final chapter I discuss the connections I make between the study and the existing body of literature, as well as implications for professional learning practice and future research.

CHAPTER TWO: LITERATURE REVIEW

To organize the review of the literature of teachers' professional learning, I begin by briefly examining the constructivist underpinnings of current mathematics education literature. I then move into the literature that describes mathematics-for-teaching. Finally, I provide an examination of the literature of the processes and conditions of teachers' professional learning.

Constructivist Underpinnings of Current Mathematics Education

Until the 20th century, education was viewed in a very linear fashion. McNergney and McNergney (2004, p. 147) outline a traditional epistemology of learning that valued knowledge as perfect ideas and universal truths knowable through expert guidance. Drake (1997) states that the traditional model of teaching was underpinned by "a philosophy that valued efficiency, productivity, and standardization... that stressed that there existed [a single] most effective 'method' for learning. ...What [was] worth knowing [were] the objective facts. Memorization assured student success" (p. 41). Knowledge was believed to be acquired by the teacher and then transmitted to the student.

Philosophies that emerged and then languished in the early 20th century valued a constructivist perspective of knowledge creation in which "exploration, questioning, and critical thinking enable students to discover or construct and use knowledge" (McNergney & McNergney, 2004, p. 147). When, in the 1950s there was concern that students graduating from high school lacked the necessary proficiency to become capable mathematicians and scientists (Klein, 2003), interest in constructivist epistemology was renewed. This constructivist perspective formed the foundation for present-day

mathematics instruction. From a constructivist perspective, learning is described by Drake (1997) as being “a process where the learner constructs new meaning within the context of what he or she already knows” (p. 41). This perspective opens the door to many different ways of knowing.

To mathematics educators, constructivism provides an explanation why student achievement is low; many students are simply unable to remember all of the facts that the traditional approach to teaching mathematics demanded (Van de Walle, 2001, p. 4). In constructivism, students’ understandings are built on what they already know so that ideas are connected conceptually and therefore can be reconstructed when needed. Instruction in mathematics consists of teachers providing students with experiences intended to build upon their existing body of understandings first and then giving students opportunities to develop skill in applying new knowledge in new problem situations. Mathematics education research since the 1960s has focused largely on uncovering likely pathways for student learning (Small, 2008).

The emerging sociology of social constructivism also influenced mathematics education research. Arguing that since all human activity exists in interactions with other people, and that what we know is influenced deeply by our social context, Berger and Luckmann (1966) state: “The sociology of knowledge understands human reality as socially constructed reality” (p. 211). They contend that our interactions with others, our culture, the organizations in which we operate, even the language we speak all contribute to the way that we think about our world. As we form schema (plausibility structure) about how things fit together (subjective reality), both as individuals and as members of a community, we inevitably come across ideas presented to us through conversations with

others that challenge the schema(ta) we have created. When this happens, we rethink what we know and make alterations—and sometimes complete transformations—of that schema (Berger & Luckmann, 1966, pp. 176-177); we construct new learning.

The relatively recent science of understanding and mapping the brain supports this theory. Kluger (2009), in writing about language and the baby's brain, estimates that babies' brains can make 15,000 neuron connections. By adulthood, that number has been reduced to 10,000. Kluger speculates, "The only way to stabilize a child's main language (or for multilinguals, the main two or three) is to begin hardening the brain around familiar sounds and syntaxes, filtering out distracting ones that will not be needed" (p. 29). Kluger also reports that researchers believe that language development needs real social interaction to develop, because babies' brains respond actively to in-person contact while they respond passively to language delivered by video recording (p. 29). Both of these aspects of brain research add credibility to the idea that knowledge is constructed and reinforced in the brain through experiences and interactions in social settings.

The application of the social constructivist epistemology to mathematics education brings new ways to think about what might be involved in teaching and learning mathematics. In considering this new approach in mathematics education, Fosnot (2007a) states that "we negotiate meaning until we come to believe that we all mean the same thing... discussing our [mathematical] ideas within a cultural, social community of discourse" (p. 5). This community-based aspect of mathematics teaching and learning is now common in up-to-date publications and most teaching materials devote sections or even chapters to developing a mathematics instructional environment that is conducive to promoting discourse about mathematics with teachers and students.

Professional Learning of Content—Mathematics-for-Teaching

Timperley's (2008) meta-analysis of the research in teacher professional learning and development begins with the following important understanding: "Notwithstanding the influence of factors such as socio-economic status, home, and community, student learning is strongly influenced by what and how teachers teach" (p. 6). Because teachers' knowledge so strongly impacts student learning, teacher learning centres on learning more about both what to include in instruction and how it should be included. The remainder of this section will focus on reviewing the literature of teachers' learning of mathematics content for the purpose of teaching, called *mathematics-for-teaching*.

In working with pre-service teachers of elementary mathematics, Boerst, Sleep, Cole, and Ball (2008) focus on the development of mathematical knowledge for teaching. They articulate a framework that differentiates between the content knowledge that teachers must learn and the pedagogical content knowledge that supports knowledge of content. Boerst et al. describe the kind of work mathematics teachers need to do in order to expertly facilitate mathematics teaching and learning with examples of the following actions:

- Making correspondences across different representations of the same mathematical idea;
- Selecting the effective representations for particular mathematical and pedagogical purposes;
- Using language that matches and supports the meaning of a representation;
- Sizing up the mathematical ideas that are highlighted in different representations;

- Helping students learn to choose and use representations to explain mathematical ideas;
- Choosing specific problems (e.g., What are good numbers to choose to bring out the concept? What makes those good numbers? What's a similar or better choice?);
- Identifying and addressing students' misconceptions through error analysis; and
- Assessing and diagnosing pupils' skills and knowledge.

Boerst et al. conclude that the knowledge of mathematics-for-teaching is a complex interconnection of ideas that must be considered for teachers to effectively facilitate mathematics teaching and learning.

Anthony and Walshaw (2009) state “only with substantial content and pedagogical content knowledge can teachers assist students in developing mathematically grounded understandings” (p. 25). This knowledge is essential for teachers to recognize and then act upon the teachable moments that occur. According to these authors, teacher content knowledge allows teachers to know “what the big ideas are that they need to teach. It helps them think of, model, and use examples and metaphors in ways that advance student thinking” (p. 25). This knowledge also allows teachers to give students productive feedback and to make effective instructional decisions. Teacher pedagogical content knowledge allows teachers to effectively coordinate discussion centred around big ideas in mathematics and to make connections among topics, strands, and subjects. Without this specific kind of knowledge, the authors state that teachers “tend to structure teaching and learning around discrete concepts instead of creating wider connections

between facts, concepts, structures and practices” (p. 25). Teaching through big ideas therefore allows teachers to integrate concepts into larger conceptual frameworks.

Fosnot and Dolk (2002) explore the interconnectedness of mathematical ideas. Unlike the traditional approach, Fosnot and Dolk find students’ mathematizing of their world to be complex and web-like. They call this interconnection *the landscape of learning*. Not only is the acquisition of skills and concepts complex, but individual students’ pathways through any given landscape is also complex, with pathways that are only partially predictable and that sometimes double back as students revisit ideas and consolidate learning. Fosnot and Dolk propose a landscape of learning for each organizing topic: one landscape for addition and subtraction; one for multiplication and division; one for fractions, decimals, and percents; and one for algebra, proposed more recently by Fosnot and Jacob (2010). Additional topical landscapes are in the development stages. From their research Fosnot and her colleagues identify big ideas, strategies, and models that contribute to a deep conceptual understanding of the topics over the course of many years of student learning. A significant portion of their research also goes toward developing a program of teacher learning about the elements of the landscapes so that teachers construct their own understanding about these topics.

Other research of significance to the development of content knowledge around mathematics instruction is the collaborative work of Smith, Bill, and Hughes (2005). These researchers have developed a protocol for considering the mathematics-for-teaching needed when considering lesson preparation, which they call the Thinking Through a Lesson Protocol (TTLP). The TTLP provides an extensive list of ideas teachers should ideally keep in mind as they consider how best to facilitate students’

learning in mathematics. This framework includes a complex set of teacher capacities to be involved in the effective planning for the teaching and learning of mathematics.

As teachers build on students' understanding and the ways that they think about mathematical situations, they build a complex repertoire of questioning skills to assess and to extend students' learning. Earl (2003) advocates for assessment to act as "the mechanism for deciding what to do to push [students'] learning forward" (p. 87).

Anthony and Walshaw (2009) also acknowledge that questions are a powerful way to assess students' thinking. They assert that

a key indicator of good questioning is how teachers listen to student responses. Effective teachers ... know that a wrong answer might indicate unexpected thinking rather than a lack of understanding; equally, a correct answer may be arrived at via faulty thinking. (p. 18)

Boaler and Humphreys (2005) enumerate nine different kinds of questions that mathematics teachers might ask and argue that the most important kind of questions call for students to explore mathematical meanings and relationships. These rarely asked questions orient students to the central mathematical ideas (pp. 37-38). Small (2008) calls these *big ideas* and focuses her work not only on questioning, but also posing good questions guided by big ideas. All of these authors consider that an understanding of mathematics-for-teaching underpins effective questioning. Effective questioning is the key to assessing and being able to differentiate instruction for all learners.

While mathematics education was evolving in North America, researchers examined the data from the Trends in International Mathematics and Science Study (TIMSS) that compared mathematics and science achievement among students in 41

nations. Stigler and Hiebert (1999) were interested in the data showing that Japanese students (among others) significantly outperformed their American counterparts in standardized tests of mathematics. One component of TIMSS was a video study that compared the teaching of eighth-grade mathematics in Germany, Japan, and the United States. Geist (2000) and Stigler and Hiebert participated in the analysis of the data and concluded that not only did Japanese teachers teach through problem solving as recommended by the reformists, but they had some compelling classroom practices that were engaging their teachers and students in significantly greater levels of critical thinking in mathematics.

According to Takahashi (2006) and Takahashi and Yoshida (2004), it is critical that Japanese mathematics lesson planning be based on teachers' understanding of the interconnectedness of the skills and concepts for the topics they teach. Both researchers also report on the way in which the development of concepts is recorded on large blackboard (or whiteboard) spaces. This public record, or *bansho*, showing the development of concepts provides a common reference point against which students can compare their thinking. The outcome of this approach is that students' thinking and reasoning was of much higher order.

As Takahashi and Yoshida (2004) and Stigler and Hiebert (1999) looked at Japanese lessons, they also observed a very different culture among Japanese teachers. Many of the lessons developed in their curriculum had been studied multiple times with collaborative groups of teachers in a process called lesson study. Following lesson-study protocols allows many groups of teachers to study not only how students construct mathematical ideas, but also the kinds of pathways they are likely to take in skill and

concept development. The lesson-study teachers observe and learn about the different strategies students might use as they construct mathematical ideas and incorporate these learnings into their lessons. They also are able to focus attention on the ways in which their questioning facilitated students' deeper thinking and reflection on the mathematics in lessons. Using the students' responses then gives teachers information they can use to decide upon the next instructional steps. In this collaborative culture, critical thinking is done by both students and teachers.

Kubota-Zarivnij (2007) states that Ontario bansho emerged from the study of Japanese lessons. It is being practiced in Ontario as an instructional strategy in mathematics that supports the continually evolving work of mathematics education. In professional learning situations, teachers work through problems in collaborative groups. They are able to use their knowledge of strategies, models, and contexts to develop a range of solutions. Augmented with contextual discussion of the interconnections of the strategies, these solutions are posted and students are directed to notice how their solution is explicitly connected to the conceptual development of ideas within the lesson; all of the thinking of students is honoured. The bansho then serves as a "collective thinkpad" (Kubota-Zarivnij, 2007) of the analysis of the mathematics, the subsequent organization of strategies, and the annotation of mathematical representations. Teachers gain professional knowledge about mathematics-for-teaching when they use Ontario bansho because it provokes both teachers' and students' mathematical thinking to be explicit when solving problems through the discussion, organization, and annotation of student work.

Mathematics pedagogy is an important element of mathematics teaching and learning. *A Guide to Effective Instruction in Mathematics K-6* (Ontario Ministry of

Education, 2006) devotes a 60-page chapter to these pedagogical issues. Based on research, the Guide covers topics such as developing a mathematical community, arranging and organizing the classroom, choosing and using classroom resources, and structuring a primary or junior mathematics class. These topics describe the many aspects that form the pedagogical part of knowing mathematics-for-teaching.

Both knowing about and knowing how to enact this knowledge in the classroom is part of a complex pedagogy in which knowledge of mathematics-for-teaching is critical. As Ball and Forzani (2007) state, “Knowing about and understanding teachers, learners, content, or environments—or even knowing and understanding all of these entities—is not a substitute for knowing and understanding the dynamic relationships among them that constitute the core of the educational process” (p. 531). In this way, content and pedagogy are interwoven into a complex tapestry of effective mathematics teaching.

Professional Learning Processes and Conditions

Until the 1990s, staff development consisted of mathematics consultants and teachers telling other teachers the best way to tell students how to carry out standard algorithmic procedures. If it is true that learners need to construct their own interwoven understandings of mathematical concepts, then many of the mathematics teachers today have not themselves constructed these understandings. Ball et al. (2005, p. 14) assert that teachers teach mathematics the way they believe it should be taught because that is all they have ever known.

The shift in thinking away from a transmissive model of developing teachers’ knowledge toward a model of building shared and collaboratively constructed

understandings calls for a shift in language describing the process of teacher learning. The focus moves away from teachers having something done *to* them (i.e., they were developed) toward seeing teachers as professionals engaging in learning. What teachers should be doing is being described increasingly as *professional learning*.

Processes of Professional Learning

In the same way that teachers interact with students, it is also useful to teachers' professional learning for them to interact with research-based resources and with knowledgeable others. Kwakman (2003) found that teachers participated more in professional learning experiences if they involved reading and reflecting on professional reading, sharing ideas with colleagues, and improving lessons (p. 166). Bandura (1986, 1997) and Goddard, Hoy, and Wolfolk Hoy (2004) suggest that vicarious experience in which a skill is modeled by someone else can also influence the way that people learn and, as such, act as a kind of resource. By using both research and the actions of knowledgeable others as resources, teachers add other informed perspectives to the pool of ideas available to their community for further investigation.

The increased focus on student learning has renewed interest in engaging in the inquiry process with actual students as part of the professional learning experience. This is not a new concept; Sparks and Loucks-Horsley (1989) suggested that observation could play a key role in teacher learning because it may take up to 30 trials or 10 to 15 coaching sessions for teachers to adopt and internalize new teaching strategies (p. 296). Joyce and Showers (1988, as cited in Sparks & Loucks-Horsley, 1989) found that after learning about new teaching strategies, "in-classroom assistance in the form of peer observation and coaching is critical to the transfer of more complex teaching skills" (p.

302). In other words, teachers learn about teaching through teaching; others' observations enhance that learning considerably.

The literature points to several ideas that influence teachers as they decide whether making changes to the ways in which they teach are worthy of consideration. Concerned with changing teachers' beliefs, Guskey (1986) proposed a model for staff development that suggested that teachers would change their beliefs only when they saw positive results in their students' learning. Lloyd and Anders (1994) found that research-based practices failed to be adopted by teachers because the practices were decontextualized from their practice, the teachers' beliefs were different from the theory of the research base, and the practices did not actually address the things that the teachers were concerned about in the first place (p. 78). In other words, teachers need to see the changes as relevant to their practice. Richardson (1994) conducted case studies showing that exposure to in-class research was the catalyst for teachers shifting their beliefs, resulting in changes in their practice. She concluded that staff development should interweave three elements: teachers' beliefs about the learning process, formal theoretical research frameworks, and alternative classroom practices (p. 103).

Many researchers describe and point to the effectiveness of classroom-based collaborative inquiry processes to learn the complex art of teaching. Sagor (1992) looks at collaborative action research, involving cycles of teacher-initiated inquiry situated in classroom practice. Stigler and Hiebert (1999) describe lesson study, a process originating in Japan that engages groups of teachers in planning and delivering a lesson, then changing the lesson based on the observations the teachers make about students' learning during the lesson. Once the changes are made, the revised lesson is taught to

another group of students; the teachers subsequently reflect on the impact on student learning of the changes. Mitchell and Sackney (2000) and Dufour, Eaker, and Dufour (2005) consider professional learning communities to be the underpinning collaborative structure in which groups of teachers examine their practice together to test theories of action in order to solve problems of their practice. West and Staub (2003) proffer content-focused coaching, an approach that uses peer relationships to hone mathematics-for-teaching instructional strategies while engaging in the act of teaching. This collaborative work allows teachers to engage in what Curry and Killion (2009) describe as the micro-level of professional learning, experiencing learning that promotes the transfer of macro learning about teaching and learning into actual practice in the classroom (p. 62). In short, the cycle of teachers asking each other questions of practice, of learning more about the research being done in that area, of putting the research into practice with students in classrooms, of examining and reflecting on the results of the new practices, and of making necessary changes and determining next steps according to the feedback from the inquiry is now ubiquitous in professional learning literature.

Another important process of teacher learning involves the role of schema and cognitive dissonance. “Schema” is the mental framework individuals develop in order to make sense of the world. Earl (2003) states that people need a rich base of knowledge about the subjects under consideration and a great deal of experience to become comfortable with the ideas and to create the mental models that organize them (p. 35). From their own experiences learning math, teachers have developed personal schema around what it is to learn and to teach mathematics. This schema will not change unless something causes them to reexamine that schema.

“Cognitive dissonance” is a term that refers to the discomfort felt at a discrepancy between what is already known or believed, and new information or interpretation. It occurs when there is a need to accommodate new ideas, and it may be necessary for dissonance to develop so that we consider them (Atherton, 2005). Mitchell and Sackney (2009) borrow from Capra’s (2002) perspective on living systems, using the term *meaningful disturbances*: “The process [of building learning communities] is powered by disturbances that capture the attention of certain people at certain places and times, and by responses that are meaningful and purposeful in and for that particular context” (p. xi). In other words, it is the disturbance in what teachers believe to be true in the context in which they do their collective work that causes them to inquire and to act professionally and to make meaning from the results of their actions.

It is possible for teachers to listen to a wealth of new ideas in mathematics instruction and fit those ideas into their own schema of mathematics teaching and learning without those teachers actually understanding them. This occurs because the new schema that underpins the new ideas necessitates a profound adjustment to their existing schema. Hiebert (2008) shows that although U.S. teachers used rich problems in their instruction, they taught them in didactically traditional ways, telling students how to solve the problems using the algorithms the teachers gave them. He states that this finding reflects the need for teachers to re-schematize their thinking about the benefits to mathematical sense-making that the solving of problems affords the learner.

Change in schema allows learning about mathematics teaching and learning to occur. Bandura (1986) theorized that learning must be flexible and be built upon generative conceptions so that people can adapt their schema about situations to new ones

as they arise (p. 111). Earl (2003) states that people may be resistant to changing their schema; “people tend to strive for relative stability between their internal conceptions and new information. They may even avoid conditions that disrupt the way they see the world. The challenge is to move beyond dissonance into productive learning” (p. 35).

One way teachers may be influenced to change their schema is by seeing someone else model effective instruction. Goddard et al. (2004) suggest the vicarious experience may allow those learning new skills to visualize themselves doing similar actions. This visualization may act as a mediator or bridge to new practices.

Finally, many teachers suffer from math anxiety, a state in which teachers’ lack of mastery of mathematics as students negatively influences their present schema. This negative schema applies not just to doing mathematics but also to the teaching of the subject. Gresham (2008) shows that this lack of confidence negatively impacts teachers’ mathematics instruction and their beliefs in the effectiveness of their instruction of mathematics (p. 13). Effective professional development facilitators need to find respectful ways to uncover the fact that teachers need to realign their schema about mathematics and teaching mathematics.

Conditions of Professional Learning

While the literature points to processes that move professional learning forward, there are also conditions that underpin and give professional learning necessary support. It is important to pay attention to these conditions of learning so that professional inquiry processes may move along without impediment.

A sense of ownership of the problem and its solutions is an important condition of learning for both students and teachers in the literature. Kline (2008) declares that

ownership positively impacted her students' learning environments by allowing them to think for themselves, to use their own struggles productively, and to embrace their own natural inquisitiveness (p. 151). Enghag and Niedderer (2007) affirm that student ownership to decide how problems would be approached and represented, both individually and in groups was significant to students' learning (p. 633). Carter (2008) reports that her students began to see that their effort moved them through the mathematics problem solving process and that they were more comfortable engaging with mathematical ideas without having immediate answers when she gave them ownership of their own problem solving (p. 137).

Teachers also thrive when given ownership and choice in their learning. Thibodeau (2008) attributes teachers' substantial and sustained changes in instructional practice to teachers being closely involved in shaping their own learning experiences in terms of both structure and operation of their collaborative study group (p. 63). Kaser and Halbert (2008) found that teachers' change of mindset toward deep learning could be achieved through several strategies, the first of which was that learners should be in charge of and own their learning: "teachers and learners benefit when respectful environments of inquiry, evidence, and choice are part of school culture" (p. 59). In their study of a teacher-directed lesson-study group, Flynn, Hedges, and Bruce (2009) describe teacher growth that expanded in a ripple effect to affect a more collaborative school culture. According to the school principal, "lesson study is 'giving the message that teachers have ownership, and that's how you're going to be a better teacher and that's how kids are going to improve'" (p. 15). Teachers who have a sense of ownership and a

belief that their teaching impacts student achievement attribute their action and their changes in practice to be the cause of the improvement.

Attention to these affective conditions supports both teachers and students as learners. Csikszentmihalyi (1990) describes truly enjoyable learning as that in which people experience a sense of *flow*. During flow experiences, learners experience a markedly heightened focus of attention and the suspension of the sense of the passing of time as they become completely absorbed in the pressing demands of a task. Distracting thoughts disappear and there is a sense of order that emerges from the struggle. As Csikszentmihalyi notes, “What people enjoy is not the sense of *being* in control, but the sense of *exercising* control in difficult situations” (p. 61). The underlying condition for this kind of experience is that “challenges [be] well-matched to skills [so that] there is little opportunity for the self to be threatened” (p. 63). If this balance is achieved, then learning will bring forth energy and excitement (Mitchell & Sackney, 2009, p. 7). Learning is an enjoyable process.

Bandura (1986) introduced the concept of self-efficacy into social cognitive theory. He defined the concept as making future-oriented judgments about capabilities to organize and execute the courses of action required to produce given attainments in specific situations or contexts (Bandura, 1997, p. 3). He states, “A capability is only as good as its execution. The self-assurance with which people approach and manage difficult tasks determines whether they make good or poor use of their capabilities. Insidious self-doubts can easily overrule the best of skills” (p. 35). It is important to the building of self-efficacy that teachers’ experiences as they are learning to teach

mathematics more effectively be ones in which they feel a good measure of success in order to drive the learning forward.

It is necessary to foster a community in which ownership of ideas and trust bring about a willingness to take risks in developing ideas as a routine part of the learning process. Mitchell and Sackney (2009) propose, “if [learners’] different dispositions and interests are honoured, and if they have opportunities to feel successful, they are likely to take risks, try, and persevere” (p. 151). If learners are able to have some choice in the things about which they learn, they will be more likely to take risks because they are able to access, build upon, and make adjustment to their own schema and knowledge base.

In order to build these collaborative processes, the literature advocates for collaborative groups of teaching professionals in which knowledge about the actual work teachers are doing with their students is socially constructed. Lieberman (1996) suggests that teacher development needs to be grounded in new ways of interacting. She proposes building new roles and relationships in which teachers act as leaders and critical friends in peer coaching situations and in opportunities to engage in action research together. To complement these new relationships, she describes new group structures within which teachers work at novel tasks such as journal and proposal writing, learning about assessment, creating standards, analyzing or writing case studies of practice, and communicating “on-line” about particular topics. Lieberman embeds all of this new collaborative work in a culture of inquiry so that

professional learning eventually comes to be expected, sought after, and an ongoing part of teaching and school life. ...Teachers who engage in these new professional opportunities often find themselves in an exciting cycle: The more

they learn, the more they open up to new possibilities and the more they seek to learn more. (pp. 189-190)

These innovative ways of engaging in teacher inquiry go beyond the transmissive approach of staff development and move toward professional learning opportunities in ways that are exciting and self-generative.

Mitchell and Sackney (2000) describe a process for collaborative work that goes far beyond working together collegially. They call to mind the processes of effective and productive dialogue, citing Isaacs' (1999) four skills of dialogue:

Listening entails deep attention to others' points of view and deep attention to one's own personal thought patterns, reactions and assumptions. *Respecting* means honoring others, legitimizing their rights and value, and being alert to what they can teach. *Suspending* causes an individual to step back and to see things with new eyes and to suspend judgments, beliefs, assumptions, decisions, preferences, opinions and so on. *Voicing* is the process of self-disclosure, where individuals open their hidden theories to others and expose the thoughts they might be inclined to keep private. (As cited in Mitchell & Sackney, 2000, p. 56)

This openness both to divergent thinking and opinions is part of a discourse community whose function is to closely examine practice, beliefs, and attitudes, thereby unseating traditional approaches and opening ways of considering novel approaches to educators' problems of practice.

The professional learning literature contends that it is critical that the processes of professional learning be supported by a community of learners. It is through the community that the conditions for optimum learning occur. Since skilled professional

dialogue plays such a critical role in the co-construction of knowledge, Mitchell and Sackney (2000) also advocate for “affective processes of invitation and affirmation [to] establish a climate within which the cognitive processes of reflection and professional conversations can occur” (p. 57). They contend that a climate of trust and caring supports the shared understandings and values of the community. This in turn is the condition that supports enhanced personal capacity and self-efficacy.

Teachers and students learn best when they co-construct knowledge through dialogue in learning communities, examining their work within the context of their work. Scott (2009) advocates for both teachers and students to engage in what she calls *fierce conversations*. She cites Gladwell (2008):

We will invite pushback, really invite it, versus going through the motions, in the genuine hope that we will be different when the conversation is over, that we will have been influenced. ...[In classrooms, teachers] create a “holding space” for lively interaction, flexibility in how students become engaged in a topic, a regard for student perspective, the ability to personalize the material for each student, responding to the questions and answers with sensitivity, and providing high-quality feedback where there is a back-and-forth exchange to get a deeper understanding. (p. 54)

Bruce (2007) states that teaching practices that emphasize student interaction improve both problem solving and conceptual understanding as students share their reasoning with one another. In other words, collaborative and dialogical processes benefit both teachers and students as co-constructors of learning.

Taking risks in thinking is one of many factors Hargreaves (2003) identifies as necessary for learning in the 21st century:

It involves deep cognitive learning, creativity, and ingenuity among students; drawing on research, working in networks and teams, and pursuing continuous professional learning as teachers; and promoting problem-solving, risk-taking, trust in the collaborative process, ability to cope with change and commitment to continuous improvement as organizations. (p. 8)

Flynn et al. (2009) also considered risk-taking as a process that evolves based on the condition of having a climate of trust. They claim:

Trust—and the growing sense of comfort that accompanied it—gave teachers the freedom not only to take risks and experiment in their classrooms, but to do so in front of their colleagues. This trust also gave them the space to self-reflect openly on their teaching practice. (p. 11)

In addition to teacher learning, Usher and Pajares (2008) suggest that the support of the community, a positive environment, and watching their peers be successful will also contribute to teachers' ability to master complex tasks, improve their self-efficacy, and therefore improve student achievement. Because there is a link between high levels of teacher self-efficacy and increased student achievement (Bruce & Ross, 2008), it is of concern that those facilitating professional learning consider collective teacher self-efficacy as important. How we engage teachers in learning about mathematics instruction matters.

Time to work together is one of the most important elements to support professional learning communities. Hansen and Mathern (2008) maintain that, next to

having a collaborative group in which to build understanding, time to teach and time to learn is the second most important condition for learning. The teachers in their study blocked out 90 to 120 minutes for mathematics instruction because “inquiry and analysis does not occur in a forty-five minute period, and teachers recognized that when they were pressed for time, they went back to a direct-instruction format” (p. 166). The Expert Panel Reports on mathematics education in Ontario (Ontario Ministry of Education, 2003, 2004) assert that time set aside for teachers’ ongoing professional learning of mathematics-for-teaching is essential. More recently, Pascal (2009) and Hord (2009) call leaders to ensure adequate time for teacher collaboration as part of the professional learning culture in order for them to learn how to help all students learn mathematics more effectively.

The more teachers learn about precision of their instruction, the more able they are to intentionally impact the learning experiences they provide for their students. Darling-Hammond, Chung Wei, Andree, Richardson, and Orphanos (2009) state that teachers’ collaborative work must be ongoing because “sustained and intensive professional [learning] for teachers is related to student achievement gains” (p. 43). They advocate for sufficient time for teachers to engage in this collaborative work because of the effects that sustained opportunities to collaborate have on student achievement. Citing Yoon et al., (2007), Darling-Hammond et al. report that when professional learning was done for over 49 hours in a year, student achievement was boosted by over 21% while 5 to 14 hours of teacher learning time showed an effect on student learning that was not statistically significant (p. 43).

Many studies show that it is effective leadership which ensures that the optimum conditions for collaborative professional learning occur. Hord (2009) advises formal and informal leaders to work toward the shared purpose of improved student learning. Kaser and Halbert (2008) claim that leaders who are growth-minded are inquiry-oriented and pursue equity and quality outcomes passionately. They know how to use evidence about learner engagement and can assess research knowledgeably. They have a sustained interest in pursuing deeper forms of learning and are constantly aware of building trust. They are willing to share leadership and to invest plenty of time to support collaboration because these leaders recognize that teaching is complex work (p. 56). In a sense, supportive leadership underpins professional learning in that leadership is necessary to advocate for and support these collaborative processes and conditions.

Citing Morimoto (1973), Richardson and Anders (1994) describe what can happen to learning communities when these supportive leaderships do not exist:

When change is advocated or demanded by another person, we feel threatened, defensive, and perhaps rushed. We are then without the freedom and the time to understand and to affirm the new learning as something desirable, and as something of our own choosing. Pressure to change, without an opportunity for exploration and choice, seldom results in experiences of joy and excitement in learning. (p. 200)

They propose a model of non-traditional collaboration with teachers in which staff developers and leaders act as one of many participants having expertise (pp. 205-206, 211). Hargreaves (2007) cautions against authoritarian modes of staff development

conducted by leaders in positional authority who decide what is to be learned and who enforce compliance. In this kind of authoritarian leadership style,

staff developers become overbearing supernannies—they watch over every aspect of teachers’ performance, they use literacy coaching to ensure compliance with prescribed programs, and they put any teacher who criticizes the scripted programs or test-driven environments on the “naughty chair” of unsatisfactory performance. Learning is reduced to training, walk-throughs become walkovers, and initiative accompanies integrity through the exit door of professional withdrawal. (p. 37)

Even though it is tempting to try to make all teachers comply with even the most noble of intents, the resistant behaviour that results is counter-productive.

When teachers react negatively towards leaders who genuinely try to set a collaborative culture, Garmston (2009) advocates for conversations to occur in which tensions surface in a safe way. She calls this process “naming the elephant in the boardroom” and says that it ought to be seen as an opportunity to gain useful feedback for leaders to solve problems. Once leaders have the information about the real problems being faced, they can make changes to the conditions they set for collaborative learning.

Most recently, professional learning researchers advocate for the stance that professional learning is complex. It involves acknowledging processes and giving them room to grow. This means acknowledging the complexity of teaching and “expanding the space of the possible and creating conditions for the emergence of the as-yet unimagined. Teaching is not about prompting a convergence into preexistent truths, but is about divergence into interpretive possibilities” (Davis, 2005, p. 87). This means that

learning which honours the complex nature of learning, whether that be professional learning for teachers or classroom learning for students, is a process of moving away from linear, preset models of traditional thought, toward a system that allows students and teachers to “interactively (re)construct and negotiate mathematical understandings within the situated ecologies of classrooms and communities. Attention would be focused on students [and teachers] making sense of mathematics in ways that are meaningful, flexible, and connected to their sense of self” (Ellis, 2008, p. 1350). Davis and Simmt (2003) call for both sufficient organization and sufficient randomness to allow for this kind of flexible response on the part of those who plan for learning. They state that most often this kind of instructional decision-making will be negotiated *while teaching*. Mitchell and Sackney (2009) submit that high-capacity schools embrace complexity and notice what they call *compelling disturbances*—those incongruities that seem to stand out from what was expected—and build responses that are meaningful within the contexts of the work in which they are engaged. In this way, professional learning is respectful and enabling, exciting and energizing, and full of possibility to effect and sustain deep educational change (pp. 194-198).

Chapter Summary

This chapter reviews the literature concerning mathematics-for-teaching and the processes and conditions that contribute to the development of meaning in professional learning experiences. The goal of this study was to apply this knowledge to the study of teachers who have effectively implemented Ontario bansho within their communities of practice, and in the process, constructed deeper knowledge of mathematics-for-teaching. This study is intended to develop a model of professional learning whose elements can be

practically applied in the design of effective professional learning initiatives. Having examined the literature on content, process, and conditions for professional learning, the next chapter will describe the methodology for the investigation.

CHAPTER THREE: METHODOLOGY AND RESEARCH DESIGN

This study set out to examine the ways in which teachers' implementation of Ontario bansho within communities of practice informs a model of professional learning. This chapter will present the methodological decisions for the research design, site and participant selection, and data collection and analysis. It also includes a discussion of the way credibility was established, describes the assumptions that were made, and enumerates the scope and limitations of the study. The chapter will conclude with a discussion of the ethical considerations of the research.

Research Design

This study was positioned within the epistemological philosophy of phenomenology with an interpretive methodological approach. Smith (2008) states that "phenomenology leads from conscious experience into conditions that help to give experience its intentionality" (p. 1). Kvale (1996) writes:

Phenomenology is interested in elucidating both that which appears and the manner in which it appears. It studies the subjects' perspectives on their world; attempts to describe in detail the contents and structure of the subjects' consciousness, to grasp the qualitative diversity of their experiences and to explicate their essential meanings. Phenomenology attempts ... to make the invisible visible. (p. 53)

These purposes mapped well onto this study, which was intended to examine and to make visible the ways that individual teachers navigated through the phenomenon of teaching using Ontario bansho within their communities of practice, and thereby added to their knowledge of mathematics-for-teaching.

Neuman (2000) states that an interpretive approach to research methodology allows researchers to “learn the personal reasons or motives that shape a person’s internal feelings and guide decisions to act in particular ways” (p. 70). An interpretive approach honours individuals’ construction of meaning and their everyday theories about their lives in constantly shifting social conditions (Neuman, 2000, pp. 72-73). This approach was ideal for this study as a means to glean the reasons why teachers decided to teach mathematics using Ontario bansho, what, from their perspectives, they learned from doing it, and the enabling conditions and processes that supported their professional learning.

Neuman (2000) also states that in an interpretive approach researchers study “people who construct meaning and create interpretations through their daily social interactions” (p. 72). This approach therefore was useful to uncover the nature of the social situations in which teachers’ knowledge was constructed.

This study followed a qualitative research design, which enabled a detailed understanding to emerge from the text about the central phenomenon of developing knowledge of mathematics-for-teaching through teachers’ use of Ontario bansho within their communities of practice. A qualitative design allowed the participants’ reflections and schemata about the meaning they made through their teaching experiences to form an emerging understanding of the processes and conditions that contributed to their increased knowledge of mathematics-for-teaching.

Site and Participant Selection

The choice of a qualitative research design requires that participants and site selection be “based on places and people that can best help us understand our central phenomenon” (Creswell, 2008, p. 213). For this study, only teachers of elementary

grades who had been exposed to Ontario bansho because of their involvement in communities of practice within a professional learning network were considered. These teachers had voluntarily implemented teaching mathematics using the instructional strategy of Ontario bansho at least once. The study was designed to involve 6 participants in all, with varied levels of implementation of bansho, in order to provide a basis for within-group and cross-group comparison and contrast.

While the participant selection was primarily purposeful, the selection of participants was also convenient in that they were drawn from the local school board. A letter of invitation to participate in the study was issued en masse to the various groups of teachers. Those teachers interested in participating were requested to reply via e-mail, indicating the approximate number of times Ontario bansho had been used as an instructional strategy and whether Ontario bansho was a strategy regularly used in the classroom. From this information, 6 participants were engaged.

The participants included 6 female teachers from one district school board in Southern Ontario. All 6 participants were voluntary members of a professional learning network within the school district. Each participant belonged to a smaller professional learning group called Supporting Understanding in Mathematics (SUM) consisting of between 4 and 12 members. At the time of the study, 5 of the participants taught in the primary division and 1 taught in the junior division. Four participants had participated with their SUM groups for 3 full years while 2 participants were in the second year of working with their groups. All 6 participants were generalists who taught mathematics but none of the teachers had any additional qualification in mathematics.

Although I had hoped to enlist teachers from different levels of bansho use, there was not as broad a range of use as anticipated. This happened because the teachers had all actively been using Ontario bansho since the beginning of the school year, both as part of the collaborative work being done in their professional learning groups and independently in their classrooms. Only one male responded to the invitation to participate but he had not instructed using bansho and therefore was not considered for the study.

Data Collection

In this study, qualitative data were collected using two sets of semistructured interviews. Kvale (1996) advocates for using the interview as an effective method to gather qualitative data because “interviews are particularly suited for studying people’s understanding of the meanings in their lived world, describing their experiences and self-understanding, and clarifying and elaborating their own perspective on their lived world” (p. 105). Interviews allowed a way to gather participants’ insights into their understandings of their professional learning processes.

From an initial review of the professional learning literature, a conceptual framework was developed which was used as the basis for framing the first set of interview questions. *Communities of practice* formed the base of the initial framework and supported the two aspects of teachers’ classroom work: *pedagogy*, and the making of *connections and coherence among mathematical conceptions, strategies and models, context and reasoning*. (This conceptual framework is depicted graphically in Figure 1.) Interview questions were written that directly linked to one of the elements of the framework. In addition to these questions, questions were designed to elicit the participants’ values and theories of practice concerning their professional learning

experiences. In this way, the questions for the interview were expressly tied into the purpose of the study. These questions formed the first interview guide (see Appendix A).

The interviews were recorded on both audio tape and a digital video recording device. Kvale (1996) states that “the interviewee’s lived meanings may be immediately accessible in the situation [of the interview], communicated not only in words, but by tone of voice, expressions, and gestures in the natural flow of a conversation” (p. 125). He points out that these nonverbal data may also prove to be important to study. Whereas an audio recording is the traditional means of recording an interview (Creswell 2008, p. 238), digital video is being used increasingly in educational mathematics research (Boaler & Humphreys, 2005; Carpenter, Franke, & Levi 2003; Fosnot, 2007a) to disseminate the practical aspects of moving theory into practice through a shared experience with the audience. In addition to the video and audio data, I recorded my thoughts and reactions as field notes following the interviews and during the analysis of the data. These data were combined with the data from the interviews at the data analysis phase of the study.

Mutually agreeable times were set up with the participants for the first 90-minute interview, which took place in locations convenient to the participants: in the teachers’ classrooms, central board sites, or private residences, thus making the teachers feel as relaxed as possible. The choice of the teachers’ own classrooms, homes, or places where we had done shared work contributed to creating this safe ambiance. The classroom locations also enhanced the richness of dialogue through close proximity to any artefacts that might serve to illustrate the teachers’ thinking. Because doing *bansho* involves using students’ work extensively, some teachers referred to the work of their students, although the student work itself did not become part of the data collected.

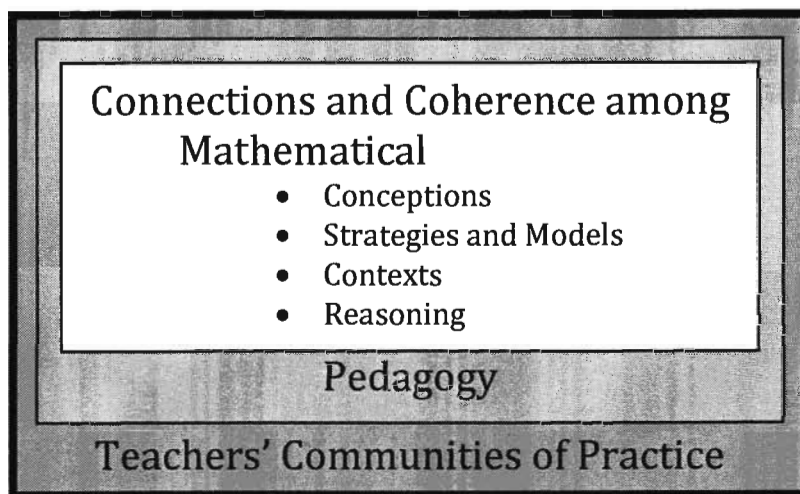


Figure 1. Initial conceptual framework for the first interview.

The data were processed in two steps. The audio recording of the first participant's interview was transcribed by an assistant who only had access to the audio recording. I transcribed the remainder of the interviews because it was more expeditious and led to a deeper understanding of the data.

In the second step, I began to add to the transcripts any nonverbal information from the video recordings that might add clarity to the intended overall meaning of the interviewee, such as illustrative gestures or emphatic use of gestures that were not accompanied by cues of intonation; however, I did not find the review of the video footage to add enough new information to warrant the time necessary to persist. Once transcribed, the participants' transcripts were sent to them for approval and they were invited to make changes to what they said or to make clarifications about what they said.

From the analysis of the data from the first interview, I shaped categories and an emergent theoretical framework. Questions for the second interview were then based on this emergent framework through what Charmaz (2000) refers to as *theoretical sampling*: "We use theoretical sampling to develop our emerging categories and to make them more definitive and useful. Thus the aim of this sampling is to refine *ideas*" (p. 519). The questions for the second interview were aligned with the emergent framework and were used to fill in any conceptual gaps of the framework. The second set of questions also allowed the participants to confirm, clarify, disagree with, or build upon the ideas from the new framework. These questions were significantly more difficult to answer than those in the first set, so I e-mailed them to the participants so that they could consider them prior to the second interview. The questions for the second interview can be found in Appendix B.

The second sets of 90-minute interviews were conducted approximately 3 months after the first and took place in similar locations to the first set. These interviews were recorded in both audio and video formats. I transcribed five of the interviews and the sixth was transcribed by an assistant. The participants approved these data and also approved the synopses made from the data. These data formed the full data set.

Data Analysis

The data were analyzed through an interpretive framework to generate useful insights about how the participants developed meaning and developed their knowledge of mathematics-for-teaching through the use of Ontario bansho within their communities of practice. This approach was well suited to this study as a means of honouring the individual participants' thinking, experiences, and meaning-making while examining the data to find patterns, themes, and characteristics that were common to the participants' mathematics teaching experiences.

Qualitative research allows for individuals' thoughts, feelings, and theories about given situations to be compared and contrasted with those of others experiencing similar phenomena. Creswell (2008, p. 243) states that the analysis of qualitative data requires the researcher to make sense of text and images to form answers to research questions. Following the collection of data, he suggests organizing, exploring, and coding the data to find and describe themes that can be validated. These three techniques were used throughout the analytic process.

An initial analysis happened during the first interview as the participants reflected on their experience teaching bansho. This *in situ* analysis (Kvale 1996, p. 190) yielded participants' own theories about their learning as they did Ontario bansho with their

students. They also made statements about the conditions that enabled them to learn about bansho and about mathematics teaching and learning in general. Even at this early stage, similarities began to emerge in the conditions that helped them to learn more deeply.

Kvale (1996) suggests that the data analysis be composed of three parts in addition to the *in situ* analysis. The first part entails *structuring* the data in a way that makes sense to the researcher. It made sense to me to keep the data organized by participant so that I could get a picture of each participant's responses to the questions for the within-case analysis.

The second part of the analysis, according to Kvale (1996), consists of a *clarification* of the data with extraneous material such as digressions and repetitions removed. For the analysis of the first set of data, I made a synopsis of each transcript and obtained approval that the full intent of the participant's words had been captured. This allowed me to compare the answers to the questions in a cross-case analysis. It was during the making of the synopsis of the first interview that the similarities between the participants' experiences and the meanings they attached to those experiences began to emerge. I also noted the use of emphatic language and intonation which indicated the strength of the emergent themes.

Kvale (1996, pp. 190-204) states that the third part, the *analysis proper*, may be approached in several ways to surface meaning. I began to reorganize the data synopses in the form of a matrix in which questions were recorded on the horizontal axis and individual participants' responses were recorded vertically. I assigned each participant a colour of paper and cut out parts of the synopses, placing the answers to the question in

the matrix. This allowed me to see the answers to each of the questions across the cases while still being able to locate the individual speakers. I had included time coding from the original transcripts within the synopses; this proved to be invaluable in locating quotations for the reporting of the data. From the matrix boards, I copied the participants' answers question by question, allowing me to read the answers as one piece of text and more easily to see repeated ideas.

I then combed through the original transcripts according to the ideas that were emerging. Using a computer-based search function, I searched the transcripts to locate participants' quotations that pertained to the idea from the matrix board. I copied these quotations into documents with clustered ideas, also recording the line and speaker. The result was many digital documents organized by idea with pertinent quotations and annotations.

As I was transcribing the interviews, I noticed some interesting uses of metaphor among the participants. The most compelling metaphor was the way in which the participants described their learning of mathematics-for-teaching. The verbs they used strongly suggested an underlying theme of traversing a landscape on a journey. The participants struggled, jumped on ideas, stepped out of their comfort zones, bounced ideas around, and so on. Using the search-copy-and-paste coding system I had developed to cluster ideas, I read the metaphors as a narrative; the sense of journey across a challenging terrain was compelling. This analysis surfaced the kinds of schemata the participants were attaching to their experiences.

To develop explanatory themes, I examined the participants' answers to the interview questions for recurring ideas across the cases. I clustered the strongest

statements together and gave them topic names. I further collapsed these topics, renaming them and checking back with the original statements to ensure that I had maintained the intent of the speaker within the new topic name. From this first analysis, four topical names emerged: book and expert knowledge; manipulative and spatial knowledge; in-practice knowledge; and *community*.

I looked across the data again and noticed frequent references to understandings and comments that only occurred when participants referred to their working with students. I assembled these quotations together. In reading this narrative, I observed that it was not just knowledge as a definable package that was created, but rather learning that occurred because of the participants' involvement with students. This observation shifted the theme of in-practice knowledge from a kind of content to a process.

Having reframed in-practice knowledge as a process, I renamed it *with students*. I began to consider whether the other categories of knowledge could be framed as processes. When I reexamined the themed data, I observed that the participants considered their interplay with a knowledgeable other as a kind of resource; *book and expert knowledge* was easily reframed as *using resources*. When I returned to the data to derive the processes that I had described as manipulative and spatial knowledge, I realized that I could describe this process as *manipulating ideas*. Rereading the themed transcripts as if they were narratives enabled me to reframe the themes in a way that was representative of the participants' actual processes.

The fourth theme that emerged from the cross-case analysis was that of community. Because I had considered the other three themes as processes, I reread the themed data to find evidence of active process within the community theme. This

consideration of the data showed an absence of action, and as I compared this portion to the other three, I realized that the existence of the community actually allowed the actions to unfold. I then understood community as being a supporting condition for the participants' learning.

The within-case and cross-case analysis allowed me to generate an emergent model based on themes and patterns in the data to which the emerging categories were related. As Creswell (2008) explains, "the focus is on connecting categories and emerging theory, not on simply describing categories. In the end, the researcher builds a theory [focused on] the relationship among categories" (p. 438). As I was developing an emergent theory of action, I spoke with several colleagues who act as knowledgeable others in my work. I reasoned that if I believed that we construct knowledge through dialogue, my conversations about the themes that were emerging in my research might help me to better understand the emergent framework I was looking for in order to ask the second set of interview questions.

Two of these conversations are of note. The first significant one occurred while speaking with my colleagues at work. We were engaged in debating what we were going to do with our SUM mathematics collaborative learning groups. I suggested that I had found some things that had been very useful to the participants in my study. During the conversation that ensued, I heard myself talk about the three processes and the way these three processes were all supported by the participants' strong need for safety within the community of learners. I saw the three elements as three pieces of fruit being held in a bowl.

I rejected this initial conceptualization; the fruit image was not effective because the skin on the fruit formed distinct boundaries. Because ideas were being represented,

an image that allowed ideas to interact with one another was needed. The image was revised to represent the community as a textured canvas on which the three primary colours were mingled together. The three colours were still recognizable, but had parts around the edges in which colours representing ideas co-mingled. I painted and photographed this image.

It was at this point that the second significant conversation occurred. I met with a knowledgeable colleague to discuss my research and emergent framework. Very familiar with my study of and work in mathematics professional learning and with conducting qualitative research, this colleague suggested I craft questions to find out the ways in which the elements of the emergent framework allowed the participants to think differently about their professional learning. A key comment during the meeting was, “Everyone knows that people need to be safe. That doesn’t add anything new to the knowledge base. You want to know what being safe allows them to think about in terms of mathematics teaching and learning.” This comment proved useful in designing probing questions for the second set of interviews.

The second set of questions asked the participants to think about their thinking and the ways they make use of resources, interactions with students, manipulating ideas, and the support of the community to make meaning from their professional experiences. Because the intent of the second interview was to confirm and elaborate on the emergent framework, it was easier to organize the second set of data directly into the themes of the emergent framework, and to identify data that did not fit these themes. Using the search-copy-and-paste coding system I had developed, these data were organized into digital

documents organized by like ideas. I also looked across the full data set to see how the emotion generated by the participants reflected the themes and confirmed them.

Kvale (1996) describes the analysis of a second interview as *self-correction* for the framework (p. 190). There were three important ways this self-correction emerged. The first correction related to the language describing the way in which the participants manipulated ideas. The participants described the connections they made using very strong visual images and referred to the schema they were building about teaching and learning mathematics. From this analysis, I realized that the process was one of making and adjusting mental maps of connections and interconnections. This also was a strong process in the research, so this process was named *visualizing and schematizing*.

A second correction arose from the first. The participants were not using resources as a condition of learning, but rather the participants' understanding was being informed with resources. They considered information they gained from watching a knowledgeable other do a bansho as a resource that was informing their development of meaning. With respect to the students, it was actually the process of *engaging with the students* that contributed to the teachers' understanding. These processes were renamed *informing with resources* and *engaging with students*.

The third correction to the framework came when I considered the quantity and emotional strength of some of the data. These data indicated that not only was community a condition for learning, but also a sense of ownership of the direction and actions of professional learning was a condition of learning. The teachers conveyed that ownership was also highly significant for their students' learning. For these reasons, *ownership* was included as a separate condition for learning in the framework.

Establishing Credibility

One method used to achieve accuracy and credibility of the study was through triangulation. Creswell (2008) offers this description: “triangulation ensures that the study will be accurate because the information draws on multiple sources of information, individuals, or processes” (p. 266). Triangulation among participants was achieved by gathering data from 6 individuals. It was also achieved in time by holding two sets of interviews separated by a time lapse. Another triangulation of data occurred in that they were collected from teachers with multiple experience levels of teaching using Ontario bansho for mathematics instruction.

Member checking was another method of establishing credibility in this study. Creswell (2008) describes member checking as “a process in which the researcher asks one or more participants in the study to check ...if the themes are accurate to include [in the analysis], and if the interpretations are fair and representative” (p. 267). Initial transcripts were sent to the relevant participant to review prior to the first analysis of the data. Once the transcript was confirmed as correctly representing what was intended to be said, I made a synopsis of each transcript; the synopses were also reviewed and approved by each participant after each interview. These approved data were used in the first data analysis. During the second interview, the participants had the opportunity to discuss the themes and interpretations following the first analysis of the data, and they were able to comment on or clarify further any part of the first analysis and subsequent emergent framework. Following the second interview, a copy of the resulting transcript was once again submitted for participant approval. This thesis report has been written based on the final approved data.

Assumptions

I have made several assumptions in conducting this study. I assume that students and teachers learn best in contexts to which they attach importance. Throughout this study, I have assumed, based on the research in the literature review, that teachers will learn best in situations of practice; it is in teaching mathematics that teachers see the need to find out what they can do to improve their teaching. I have assumed, also based on the literature, that teachers learn effectively within communities of practice. I have assumed that teachers care about their teaching and about improving student achievement and that they care that all students learn to the best of their ability. I have made the assumption that teachers understand that they are able to influence their students' learning through effective instruction. Finally, I have assumed that teachers are reflective and interested in changing their beliefs and paradigms if they come to understand that it will result in what is best for their students.

In discussing mathematics-for-teaching, I made the assumption that there actually is a set of knowledge and skills that can be observed and learned over time and with experience. In addition, I made the assumption that this set is not a discrete list, nor is it a set that can be completely defined; I assumed that each item will mean slightly different things to every reader, based on the personal context they bring to their understanding of mathematics-for-teaching. I also assumed that knowledge of mathematics-for-teaching cannot be exactly quantified.

While each teacher's experience is different, I assume that teachers who are willing to adopt innovative strategies such as Ontario bansho already make many of the assumptions I have made with regard to constructivism and the honouring of multiple

ways of knowing mathematics and mathematics-for-teaching. These assumptions shape the character of this research, form the basis of shared understanding, and make the results of this study more relevant to readers as it further informs their practice.

Scope and Limitations of the Study

This study focused on elementary teachers of mathematics who have learned and are using Ontario bansho in their mathematics instruction. The participant selection was not random and no attempt was made to represent the learning of mathematics teachers in general. Neither was the participant selection of sufficient size to be used to generate a model that is predictive with regard to the professional learning in which all Ontario teachers who practice Ontario bansho might engage. The opinions, experiences, and learning reported by the participants did not represent those of all Ontario teachers; conclusions cannot be drawn about all Ontario mathematics teachers based on this study, nor can the results be generalized to a wider population.

Ethical Considerations

This study followed the Brock University guidelines for research with human participants. The study was submitted to the Brock University Research Ethics Board for review and no research began until clearance from that board was received (see Appendix C). Because no students were involved in the study, approval from the board of education in which the teachers were employed was not necessary.

The teachers in the study participated by invitation. They were fully informed about the nature of the study and their level of involvement, and they were not coerced in any way to participate. There were no known risks associated with participation in this study. The teachers and I are colleagues and/or professional friends, and I was not

responsible for or involved in the participants' supervision. Prior to the study, we participated as colleagues in a network of professional learning groups in which responsibility for leadership was shared.

The teacher participants with whom I worked had already volunteered a great deal of information about their practice in public professional group discussions and had indicated informally that they might be interested in participating in a study to reflect on their experiences using Ontario bansho. Letters of invitation included a statement that the interview would include both audio and video recordings and an assessment of the risks and benefits to the participants were sent, along with a statement that they could withdraw from the study at any time or refuse to answer any questions without any repercussions.

The original video footage, the original audio recording, and the transcripts of the interviews were kept in a secure location at my home. I enlisted the help of one person to transcribe the audio recording of the interview and obtained a confidentiality agreement signed by the transcriber. I reviewed the transcripts of the audio recordings while viewing the video footage of the first two interviews but found that there was little nonverbal information deemed to be significant to the transcripts. I therefore did not view the remaining footage until I prepared to defend the thesis. I was the only person to view the video footage.

Throughout the study, the participants remained anonymous. The participants chose a pseudonym and I referred to each teacher solely using that pseudonym. The tapes and digital encoding of the data will be destroyed one year after the completion of the thesis.

Should I choose to use the video footage as part of a presentation for dissemination of the results, I will secure individual participants' express written permission prior to using any footage for this purpose. Video may prove to be a valuable tool in disseminating information about Ontario bansho, especially to show reluctant teachers potential benefits to their own practice by teaching using Ontario bansho as an instructional strategy. Because I always understood that the video footage might be used, I included this information in the letter of invitation and the accompanying consent form.

Chapter Summary

This chapter has outlined the methodology for this study. Using a phenomenological approach, two sets of interviews were conducted. The qualitative data from these interviews and field notes were analyzed through an interpretive process to identify themes and trends emerging from the data. These themes informed a model of professional learning of mathematics-for-teaching as the participants implemented Ontario bansho within their communities of practice. Chapter 4 will present the results of this investigation.

CHAPTER FOUR: PRESENTATION OF RESULTS

The purpose of this study was to investigate how teachers' implementation of Ontario bansho within their communities of practice informs their professional learning process of acquiring knowledge of mathematics-for-teaching. The results contained in this chapter were obtained through a phenomenological approach to inductive analysis of two sets of interviews conducted with 6 participants. I analyzed the data using within-case and cross-case analyses to ascertain emergent themes based on the strength of both the individual participants' opinions about the topics as well as the frequency with which the participants felt and thought in similar ways about the topics. The framework that emerged from the analysis consisted of three processes and two conditions that led to the participants' professional learning and the development of meaning. *Informing with resources*, *engaging with students*, and *visualizing and schematizing* were the three key processes; *ownership* and *community* were the conditions that facilitated the participants' learning and development of meaning in their professional learning. This framework serves as the organizational structure through which the findings are presented in this chapter.

Informing With Resources

One of the three processes in which the participants engaged to develop knowledge of mathematics-for-teaching was their use of resources. The process of being informed with resources served to add the ideas and practices of researchers and knowledgeable others for the teachers to consider as they reflected on their own work in the mathematics classroom. Their comments identified four sources for this informing

process: traditional professional development in the form of workshops, the use of manipulatives, professional reading, and interaction with a knowledgeable other.

Traditional workshops served as a springboard for the participants to begin the process of looking differently at their mathematics instructional practice. A sense of dissatisfaction with their math practice and subsequent piquing of interest in the content of traditional workshops figured prominently in Laurel's and Elaine's statements about changing the way they taught math. Laurel thought that this kind of professional development was a contributing factor to her seeking new direction in math instruction:

I felt dissatisfied with what I was doing with math.... I knew there was more to math. I had the opportunity back several years ago to become the math lead teacher and I attended several workshops at the board office. That was the connection; then an opportunity came for me to become involved as a SUM teacher (the school board's professional learning network, Supporting Understanding in Mathematics). (Interview 1)

Elaine also described learning more student-centred activities in workshops, trying them out, and then being uncomfortable with going back to traditional textbook approaches to teaching mathematics. She too subsequently became a SUM teacher (Interview 1).

Although the participants found value in some of their traditional workshop experiences, they also identified some problems. Laurel described her experience in after-school workshops:

I used to embrace going at the end of the day and sitting down and getting a little booklet and filling in things and thinking, "Oh, that's great. Just feed me." Then

I would go away and sometimes I would look back at the booklet. At the moment I was hearing it but then it got tucked away in the file. (Interview 1)

Riley identified her reservations about the value of workshops:

If you just go to workshops once in a while, there's no one to dialogue with. The only way we learn is through dialogue. When you go to a workshop or you go to another environment similar to that, you can't talk. (Interview 1)

Leigh described a school-based workshop experience as being akin to something being aimed at her:

It's *awful* and it's *so* beamed at us all the time.... What comes through is, "You need to do this now; this is the only way to do this. It's the right way to do it and if you don't do it, there's something wrong with you." (Interview 2)

In general, the participants found the traditional workshop format to be predominantly a passive experience and of limited use to them.

A second source of knowledge building with resources was the role that manipulatives played for the participants. Using physical objects seemed to allow the participants a window into seeing mathematical relationships in new ways. Cara described one scene in which she had observed someone cut an array made of graph paper into smaller arrays to demonstrate the distributive property of multiplication:

Before I started heading down this path, I didn't picture any of those things in my head.... I remember the first time I saw [someone] make an array for a multiplication problem. I was blown away! Of course I had known about an array, but I had never seen anyone make one for the kids and then turn and talk—

it was absolutely brilliant. From that moment on, I see [that person] cutting that array apart. (Interview 2)

Riley also said that she needed manipulatives to visualize new relationships: “it very much lets ideas bump into each other.... It’s just mind-boggling to see how interconnected [mathematics] really is!” (Interview 2). Laurel verbalized her experience with manipulatives and the connections they help her to make:

When I’m manipulating, it helps me. When I used those cubes that made the [concrete] bar graph and then moved them to make the circle graph, I thought, “Ah!” It completely made sense to me to physically do that manipulating. I get excited about those kinds of connections, where you can use manipulatives to hook two different ideas together. (Interview 2)

Elaine summed up her need to gain a new understanding of mathematical relationships by working with manipulatives so that she could help her students make connections:

Relationships—we were never expected to relate things before in math. We never had to have the children relate. As long as we knew the algorithms, the relationships weren’t important. When they see the relationship, that’s the connection that makes it real. (Interview 2)

Riley and Cara thought that connecting manipulatives to real-life situations was important to their students being able to communicate their mathematical thinking. Cara emphasized:

[Using manipulatives] is the *only* way for them to make the connections. The dots on the number line could be the same as the ticks on the ruler. They can more easily transfer. [Using manipulatives] makes them concrete and makes

them real—understanding that there’s so many different ways to think about and represent and methods that you *could* use to solve a problem. (Interview 2)

Riley also described the crucial role she thought manipulatives played in representing students’ abstract thinking:

The thinking that came from using the manipulatives is so essential that if they can’t move from the tactile and visual to the abstract, they need to go back.

Sometimes with the expectations, with all the pressure of getting ready for the next grade and EQAO, it’s hard to go back. Most of your kids are at grade level, but then you’ve got your little one back here who just is still—. That just crushes me. (Interview 2)

For Riley, it was disturbing for students to miss the connections to real mathematical situations that could be made using manipulatives. Cara claimed that using manipulatives “gives [the students] the ability to picture what they’re thinking. ... It just makes it real for them in the same way it did for me” (Interview 2). Cathy revealed a unique perspective with respect to using manipulatives. Although she did not make the connection to her own learning, she suggested that students were more likely to make mathematical generalizations when they used manipulatives:

Even if a child uses the same strategies but one child uses cube-a-links and another child uses two-sided counters, sometimes even that helps them. If you said to them, “Do it over again with the two-sided counters,” and they had used the cube-a-links, they would be surprised to see that it’s the same answer. So manipulating things help them to generalize. It helps them to realize that there’s

more than one way to get to this—it helps them with that generalization.

(Interview 2)

The visual aspect of manipulatives served as a resource to build understanding and to facilitate students being able to communicate their mathematical thinking.

Reading print resources also allowed the participants to add to their knowledge of mathematics-for-teaching. They noticed big ideas (Cathy & Elaine, Interview 2), strategies (Elaine, Interview 2), models (Leigh, Interview 2), and manipulatives (Elaine & Laurel, Interview 2). Elaine kept a copy of Fosnot's (2007a) landscape of learning of interconnected strategies, big ideas, and models posted on the wall behind her desk as a guide for her thinking which she referred to in her planning and teaching (Elaine, Interview 2). Laurel found that the focus on big ideas was of critical importance in her understanding of mathematics-for-teaching:

One of the things that I have learned the most is the whole idea of the big idea and how math is just filled with those big ideas. I need as a teacher to be aware of what those big ideas are. I should be able to express that and to know, as I'm beginning lessons, what exactly the big idea is that I'm aiming for and that it is the focus of that particular lesson or series of lessons. (Interview 1)

When the 6 teachers described the thinking that they engaged in when doing professional reading, they did so with duality of purpose. On the one hand, they echoed Leigh's comment that to increase their own individual knowledge of mathematics-for-teaching, "you have to be a critical consumer... to recognize when it's useful" (Interview 2). On the other hand, the teachers also indicated that they related what they read back to their individual class of students. Laurel and Leigh kept track of important points by

writing in the margins of their readings and reflecting back on them at a later time.

Laurel recounted:

I love to highlight and make notes on the side. They might even be things like a star or “really important” and that will grab my attention and bring me back to rereading a portion. Or I’ll bracket something and say, “Wow.” I’m thinking about how those highlighted things or chunks of text summarize for me what this whole learning is really about. When I find a statement or series of statements that really cut to the core of what we’re doing and why we’re doing it and the benefits to why we’re doing something—the benefits to children and to their math understanding—something really concrete that is the foundation of what we’re doing, that’s when I “wow” them. (Interview 2)

Cathy said that she kept two kinds of written records: one for new insights into mathematics-for-teaching and one relating to her immediate or imminent practice with her students (Interview 2). The other participants also made frequent reference to thinking about their own practice with their students when reading professionally. The participants echoed Cara’s statement that they were looking at the resource to answer the question, “Will this help them with their understanding and help me with my understanding at the same time?” (Interview 2).

The participants were unanimous in the style of professional reading that they preferred and the reasons for this preference. Like the others, Leigh liked to “see dialogue of class vignettes. Either it’s validating what I thought they knew or what I thought about where they need to go next. Or it’s an ‘A-ha!’ and I need to do something [different]” (Interview 2). A grassroots style of authorship helped the teachers to connect

the ideas within the reading to their practice; they visualized what a lesson might look like in their own classroom. Laurel summed up the value of this:

When you can read about the way something looks in a particular classroom and the actual questions that were asked, you can visualize what it looks like in a classroom. Then it's not a far leap to connect it to your own classroom.

(Interview 2)

The participants also articulated satisfaction about their professional reading. Cara and Leigh related their satisfaction to their practice. When asked what was satisfying about doing professional reading, Cara answered, "Validation. Excitement—if it's something I think I could bring to my kids and my classroom" (Interview 2). Elaine and Cathy found their own learning to be what was satisfying:

I like stretching. I love stretching and growing. It's fun to try and watch kids stretch. It's fun to talk about it! And watch people stretch. I like learning; it's fun! I guess stretching is synthesizing and taking what you know and adding more to it. Becoming more. Learning more. (Cathy, Interview 2)

Elaine revealed a tension between satisfaction and the need to continue to grow in her understanding:

It's satisfying, but it's also the more you get to know the more you know you don't know. It's satisfying, but you're getting into deeper waters. You're starting to read the brains that are *really* knowledgeable, and although you've advanced, you've got that much further to go. (Interview 2)

The participants all experienced collaboration in groups that included central staff who did not have classroom responsibilities and who worked only in mathematics

teaching and learning. When considering the role of these knowledgeable others as a resource, 5 of the participants referred to the necessity of having that person be a part of the safe community of learners. For Cathy, this was a critical component:

It's got to be a *safe* knowledgeable other. ...You're both working and researching together....It's got to be some knowledge coming in, where one person's at a higher level than the other person or whether you're both at a higher level in the subject matter, where you're both searching—where you're both calling each other higher. (Interview 2)

Riley said, “I feel very confident in the people I can go to and say, ‘I don’t understand this math’” (Interview 1). Even though Elaine did not expressly state that safety was important in the relationship with a knowledgeable other, she referred to needing the safety of the group in order to reveal the fact that she did not know the content well (Interview 1).

The participants were not only informed with respect to mathematics; they also looked to the knowledgeable other to help them to organize their thinking about how to conduct a bansho. Leigh valued the opportunity to see an organizational structure for the bansho: “You're looking for some sort of framework. You want some sort of framework for how it goes” (Interview 2). In addition, all of the participants spoke in the second interview about practical tips they gleaned from watching the knowledgeable other do a bansho. These tips ranged from arranging the student work in a concrete graph style (Leigh, Interview 2) to writing the big idea on the bansho after the teachers drew the conversation with the students to a close (Elaine & Laurel, Interview 2).

The participants also reported being informed by knowledgeable others about the technical aspects of conducting a bansho. Cathy used the following metaphor:

Teaching is a science and teaching is an art. I think from watching a bansho, I can get the science. I can get the technique. But I can also learn from the dance. There's a definite dance going on between the instructor and the students. The teacher is definitely the male in the dance. You can watch the dance and you can learn from the dance how much to give, when to give, when to stop them, when to start them on this, who to ask, what to ask. Some of that's the science, but a lot of that's the dance. You can learn from both. ... If you can dance, you can learn to dance better by watching someone else dance. (Interview 2)

Laurel was interested in looking at what she described as the "rhythm" of the dialogue with the students:

I'm thinking, "What would this look like in my classroom?" ... I'm thinking of the rhythm of it. There's a lot of back and forth: the question, the response; the deeper question, the response. At what point—how long do you go on? How long to let a child explain? At what point in their struggle to explain do you jump in? The rhythm of that is important in the sense that it keeps the students' attention and so that I don't take over. (Interview 2)

Elaine stated that the interplay of a knowledgeable other with students in a bansho "helps you to see more what can come out of it. Maybe before you do it yourself, to watch someone do it, is a dress rehearsal" (Interview 2). Although none of the other participants used this kind of metaphoric representation of the watching process, all of the

participants referred to many elements involved in seeing a knowledgeable other coordinate the discussion with students.

All of the participants made connections to their own students by thinking about how the discussion would be coordinated if they did it with their own class. Questioning in particular figured prominently in 5 of the participants' reflections. Elaine described the following considerations about which she thought:

How they guide the child through the discussion. ...What kinds of things are being asked or pointed out or questioned. ... Those are going into my head for the next time. They were effective; they helped and they seemed to elicit information that moved the solution further on or moved the learning further on. ... I think watching someone do a bansho you get the feeling that that's a good question, that's a big idea, that's a model and we can go further with that. That's what you're doing; you're sorting through what's coming out as you're watching and listening. It helps you to see more what can come out of it. (Interview 2)

In addition to these considerations, all of the teachers were thinking about how they would modify the lesson to best suit their students. Riley said she thought about how she could "tweak" the lesson: "I'm looking for things I don't do to improve my own when I see banshos. How can I make mine better?" (Interview 2). Cathy, Elaine, and Laurel noted thinking about the students' level of engagement as a guide to the kinds of things they should make mental note of or think further about. Cathy put this into words in the following way:

I don't know if I'm watching the *someone* so much as watching the *response* to the someone and watching the children and the level of engagement. I'm

watching where *they're* at. If the engagement is high, then I'm thinking about "What can I put in my hip pocket?" If the engagement is low, I'm thinking, "How would I approach this differently to increase the level of engagement?"

(Interview 2)

A sense of satisfaction with the collaborative work in which the participants engaged began to emerge. The second interview uncovered a pervasive sense of satisfaction that came from the teachers' feeling that the learning they had done to date was validated and that they were doing good things for their students. Cara made the following emphatic assertion when questioned about ways in which she felt satisfaction when she watched a knowledgeable other facilitate a bansho:

Validation—I know something [laughter] and I'm on the right track. My children are going to benefit from that. I can recognize the amount of learning that I've done and that is hugely satisfying, knowing that my kids are benefiting from the amount of time I'm out of the classroom. It's been a lot of work for me, too, so to have that sense that the kids are benefiting from that is hugely satisfying.

(Interview 2)

By engaging in professional reading, the participants added to their knowledge of mathematics-for-teaching. Using manipulatives contributed to both the teachers' and the students' building deeper conceptual understandings. Finally, engaging in conversations and co-teaching with knowledgeable others gave teachers insights into the organizational and cultural aspects of doing Ontario bansho. Had they not had a knowledgeable other as part of their community of practice, they may not have had access to these knowledge-building experiences.

Engaging With Students

The second of three processes in which the 6 participants engaged occurred while they worked with students. Teachers worked with students by doing banshos in their own classrooms and during collaborative sessions with their SUM groups. In these sessions, they co-planned, co-taught, and reflected together following bansho lessons. From these collaborative experiences, they added to their knowledge about mathematics-for-teaching and developed meaning, becoming more aware of the classroom culture they were creating.

The teachers constructed deeper understandings for themselves about mathematics-for-teaching as they planned for banshos by examining the work the students had produced. All of the participants described learning more about the ways students understood the mathematics through the strategies and models they used. Riley described such an incident:

I was sitting down with a colleague and we were going through the kids' work and I said, "Oh my word, did you see the way that student figured out the multiplication?" We couldn't figure out for the life of us how they had figured it out. So we had to go and ask the student. She had done *amazing* mathematics in her head! It was like "Okay! I would never have thought of that!" ...It's just the whole idea of learning from the kids. ... I've learned to respect my students' thinking processes, instead of trying to box them in. I think that's the biggest thing about the bansho. (Interview 1)

Even Leigh, whose knowledge of mathematics content was not enhanced by looking at her students' work, gained insight into the ways in which her students used models like

the number line through seeing the ways they approached the mathematics they used (Interview 2). Building knowledge of mathematics-for-teaching also was strongly evidenced by the way all of the participants filtered their work with resources through their work with students; they connected the theory with actual practice as they enacted co-constructed lessons together.

All of the participants talked about gearing their questions so that they moved the students forward conceptually. Laurel described how she thought her questioning influenced her students:

I'm very much aware of asking open-ended questions, of asking questions that will make them think. I'm not always great at it, but I'm very aware now that what I'm asking can influence, hugely influence their confidence level, their understanding, and their deeper thinking. So I have a responsibility to make sure that the questions I'm asking are actually going to get them where they need to be. Or to the next level anyway. (Interview 2)

Although everyone spoke about learning more about questioning their students, Elaine, Laurel, and Riley mentioned that this aspect of instruction had been a focus of their practice. Elaine went so far as to say, "I'm really getting proud of my questioning. I'm getting *much* better at questioning" (Interview 1). Both Cara and Laurel felt that what they had learned about questioning their students had spilled into other subject areas. Cara declared:

The questioning that I'm learning through bansho and the support that I've received for learning how to *do* that has affected how I interact with my students,

right across all the classroom areas, in language and in social studies and in science especially. (Interview 1)

All of the participants learned more about mathematics-for-teaching through the opportunity bansho afforded them to engage in assessment for learning. The more they learned about individual students' learning, the more the participants were able to build an understanding of the range of student learning within a topic. Laurel considered where her students were and used that knowledge with respect to the continuum of understanding for a given topic to know what to plan in her upcoming instruction:

They're at all different points along that continuum of understanding and of being able to communicate. That's where I come in because I need to then bring them along that continuum through the different activities and the things that I'm doing in the classroom. (Interview 1)

Riley found that this became knowledge that the children themselves could have:

With the bansho, all of [the strategies] are shown. I like the organization of it—I like when kids get to see, “Okay, this is where *I* am. And this is where I might want to go.” Or, “This is where my teachers want me to go, or where I need to go, based on expectations.” I like that organization. In other forms of direct teaching, kids don't get to see where they are on the continuum of “where do I fit in?” A bansho clearly [shows], “This is where you might be,” still valuing that we all have different ways of doing and learning. (Interview 1)

Finally, Elaine summed up the result of this assessment for learning: “Now, when you do a bansho, each *child's* learning is supported” (Interview 1).

The classroom culture developed through doing bansho also influenced the participants' understanding of mathematics-for-teaching. Laurel commented on this shift in culture: "To watch them—they're confident. Their communication—their math talk that goes on has been exciting to watch and has helped to change the tone of the classroom—the math tone" (Interview 1). Every participant was adamant in affirming Laurel's observation about her students. Cara said the following to explain this difference in tone: "I think the kids are much more invested when they're learning from each other and when they are explaining their own thinking in their own solutions because something now that they are interested in and a part of" (Interview 2). This sense of student ownership of the bansho process was one that Cathy considered critical because every child benefited:

Probably where [bansho] is successful is that you take various levels of work—whether you're a child who's struggling or a child who's soaring in the clouds as far as your math goes—that child is validated and the fact that they find their own thoughts and put their work up, even the child whose work isn't well-presented, they're still validated. But they're also stretched. Even your high flyers are stretched somewhere along the line. (Interview 2)

The ownership did not only pertain to the students' written work, but also to the conversation that happened as students justified their solutions to one another. Elaine asserted, "They're having to consider their classmates as teachers, learning from each other. That builds new social avenues with each of them" (Interview 1).

The teachers were also unanimous about the ways that doing banshos helped their students to make connections. Cathy illustrated this by recounting a scenario in which

her students made a connection between the commutative property of addition and previous instruction:

They did their little bansho ... Well, the lights came on and the $8 + 2$ and the $2 + 8$, they were switcheroos! There were connections made. We'd been doing lots of things with making 10 before that, but it was like seeing it all up there and alive and it was like, "Oh! Hey, this is cool!" (Interview 1)

Riley also spoke about making connections:

They have to connect to their previous learning. Even if it's viewing someone else's strategy that they hadn't thought about and they're thinking, "Where have I seen that before?" ... I think those kind of connections mathematically are important. (Interview 1)

Elaine felt that students' making connections through bansho was important to build their confidence as learners:

They're happy because they figured it out. They're making connections and they're seeing them. ... With their mathematical knowledge, when they make a connection, things fit in and they feel more comfortable and they feel ready to move on. Not just because somebody told them what to do, but they know what to do so they feel comfortable moving on. They feel a sureness. (Interview 2)

The assessment for learning that was done during the problems and banshos allowed the participants to focus more specifically on areas that the students needed further development. Laurel was working on helping her students to be better communicators:

They're trying to explain. So hopefully, as they gain experience, they'll also learn to better communicate what it is that they were thinking. It gives them a confidence, it gives them the vocabulary, it gives them the experience. So hopefully, some of the ones that are less capable of explaining themselves, as they continue with this throughout the year, they'll become better communicators of their understanding. (Interview 1)

Another way that the culture of the classroom was affected was that there was an increase in the level of engagement. Elaine, Cara, and Laurel said that their students were excited about their bansho work and both Elaine and Laurel described scenes in which students who were not able to be present for a bansho expressed disappointment that they had missed math (Elaine, Cara, & Laurel, Interview 1). Cathy suggested that being able to solve problems would be reflected in the children's attitudes; she described her children as being successful, engaged learners (Interview 1).

Riley felt that the discourse community that she had developed through bansho caused a respectful climate to be built in her classroom:

They talk about math; they feel that they can say whatever needs to be said to get them to understand the mathematics without someone saying, "You're stupid." Nobody says that because everybody's valued. ...In mathematics, because there's a lot of things to remember and a lot of things to figure out and understand, when that social interaction happens, that talking, the mathematics comes out. Someone will see one perspective and someone will see another. I think it just all falls into place. I think that social interaction is extremely important. (Interview 1)

This culture was one Riley believed should be prevalent in schools:

It reaffirms to kids, too, that we're all different and we can all come with what we know and still be successful. I think that culture *should* be in all classrooms. ...

The ability to communicate and to talk and to value and to question. And it's okay to question and to not get it, but not to feel threatened that I don't get it.

That kind of thing has spilled over immensely. The kids feel confident now, speaking and talking in front of people, or strangers who come in to visit. "I have something to say and it should be valued." I think bansho plays a part in it.

(Interview 1)

Being able to practice Ontario bansho with students allowed the participants to put theory into practice. Through engagement with their students, the participants were able to reflect on and make new meaning about the ways in which knowing mathematics-for-teaching contributed to their students' understanding and the classroom culture they were creating.

Visualizing and Schematizing

The third of three processes in which the 6 teachers in this study engaged in order to develop meaning occurred while they were visualizing and schematizing. In order for the 6 teachers to begin their collaborative work with their professional learning group, they had to experience a shifting of their schema about what it is to teach mathematics. Laurel described the schema with which she came into teaching:

I used to think that safe meant traditional and extremely organized and that there were certain expectations I had to fulfill as a teacher. What I did was very much the same for all the children because what I did was very cut and dry. I had to be in control and in charge because this was my classroom and my students. I was

the one that ran the classroom and did most of the speaking. ... I was told for years, when I was first teaching, how organized I was and what a good teacher I was; how well I worked with children. It constantly reinforced the traditional way. I was quite pleased with how I was teaching. ... I guess it's the tradition that you come into teaching thinking. I think that I took a lot of what a teacher should look, sound, and be like from my own teachers. (Interview 2)

Whereas only Laurel, Elaine, and Riley referred to their preexisting schema about teaching and learning math, all of the teachers spoke directly to shifts in the way that they taught math in the first interview (Laurel, Elaine, & Riley, Interview 1).

All of the participants reported that they had experienced a major shift in their thinking about mathematics teaching. For 5 of the teachers, it began with a general dissatisfaction with the way they were teaching math. Laurel said that she “figured there had to be something to make it more than just that rote learning” (Interview 1). Leigh also noted feelings of frustration, feeling both overwhelmed and dissatisfied by the rigid approach of the textbook:

When I transitioned to [a new grade] it was so overwhelming that when someone said to me that this is what we use and they handed me the manual and this is how we do it, I thought, well I have no frame of reference for [this] grade. I don't really know what they need to know. ... It got me through that first year and then as my comfort level with [the] grade grew that first year, I thought, “This really stinks.” (Interview 1)

This dissatisfaction provided fertile ground for professional learning opportunities that piqued the interest of the participants. The ideas presented in those opportunities

provoked the teachers to consider making changes to their schema about mathematics education. Laurel used the term *paradigm shift* to describe her changes; the shift sometimes caused her to feel uncertain:

I've come through this struggle and it hasn't been easy. It's been a little bit frustrating at times and there's been a sense of uncertainty as I work through all of this. But the more you do things and the more you try things, the more you rid yourself of that fear, the more you go, "Yeah—I get this! I'm understanding what it's all about." I'm coming through that uncertainty and coming out the other side. I'm still not *there*. There is no *there*. (Interview 1)

Like Laurel, Cara, Elaine, and Riley perceived of the shift as an ongoing process using navigation metaphors like journey, struggle, being on board, and experiencing a big learning curve as they continued to negotiate meaning (Laurel, Cara, Elaine, & Riley, Interview 1).

The participants also experienced a change in the culture of professional learning that translated into their visualizations of the way that they would choose to teach other teachers about bansho if they were given the opportunity. They no longer thought of teaching others in the way that professional development was done, that is in a transmissive fashion. Rather, they all described engaging teachers in a collaborative approach, co-planning lessons and co-teaching those lessons in classrooms of students. When asked how she would teach another teacher to do bansho, Cara described the way she imagined:

I would sit down with them and work on a problem that we'd present to the class and probably find the anchors and discuss exactly which big idea we're looking

for and what kind of questions might guide toward the end results. ...Then maybe after they watched for a little while, take turns and then—or co-teach and build questions together and have discussions that include the students as well.

(Interview 1)

Elaine felt that teachers new to the bansho process would need to see how “organic” teaching through bansho is (Interview 1). The participants also felt the need to pay careful attention to the safety and comfort levels of their peer(s), working on areas of mathematics instruction with which their colleagues were comfortable. Riley, who had actually taught some of her colleagues, described her theory-in-action:

I think teachers have to initiate or want to be part of it. ... If you make teachers feel judged, it just makes them want to shut down. ... The teachers [should be] involved in the process about a topic they’re doing [with their students]...instead of it being forced or contrived. ... I brought them *into it* instead of saying, “This is the way you ought to do it,” or “This is the way it should be done.” I think if you do it in a more gentle manner instead of coming in forcefully, it just turns out positively. (Interview 1)

Two metaphors emerged to visualize the way the participants conceived of the interconnection of ideas, concepts, and strategies in mathematics. Cathy and Elaine borrowed Fosnot’s landscape of learning to describe the way mathematical ideas are connected (Cathy & Elaine, Interview 1). The landscape metaphor and the movement among its elements was also reflected in all of the participants’ use of metaphoric language to describe the learning journey of themselves and their students: Leigh was glad she was not leading her students down the garden path (Interview 1); Cara said that

it had taken her a while to get to where she was in her professional learning in mathematics (Interview 1). Riley, on the other hand, preferred the model of a net:

There's an intertwining [of math concepts], an overlapping—it reminds me of a net. This part over here connects to that part, and you have to have the whole net to make it work. To me, a landscape, yes, but it reminds me of a net to make a picture in my mind. Because if there's one hole in the net, things will fall through. (Interview 2)

Riley, Elaine, and Laurel used the image of a continuum to describe the range of strategies and understandings that their students might exhibit in a bansho. Riley saw the connection among strands of mathematics and across grades to be important in the way she schematized mathematics instruction. She recounted,

You're constantly evaluating your thinking and always making connections, whether it is across the strands or a grade ahead or a grade below. I remember I went into a class and I saw that this was so important to make a connection along the continuum of grades. ... I'm constantly looking at what's in my own grade and thinking about where this will fit later. (Interview 2)

However, Cara rejected the idea of a continuum by saying, "I used to see it as a horizontal line with little points on it that you had to finish one section before you can move on to the next section. Now I don't see math like that at *all*" (Interview 1).

Another way in which the participants visualized or schematized was by manipulating ideas through discussion. Cathy saw this process in the following way: "We bounce ideas off each other. We're kind of thinkers together; growers together; learners together" (Interview 1). Cara saw the co-planning process as one in which

questioning was planned in order to build a concept (Interview 1). Elaine said, “When we get together in our math group, we’re manipulating ideas all the time; [we’re] bouncing ideas all the time” (Interview 2).

The participants also engaged in an intrapersonal manipulation of ideas. Cathy described her thought process:

You take the idea, you chew on it some more. You’re thinking while you’re doing tasks. You have time when you’re driving or vacuuming. ... You play with them a bit. It’s kind of a thinking and a doing to get them stirred in there. They have to become part of your repertoire. (Interview 2)

Riley sought to link the schema she had about big ideas to her students’ understanding by engaging in in-the-moment introspection that she called questioning the questioner:

When I’m asking students questions, I always have the end in mind—where I want my students to go. Based on that, I will question myself. It’s funny because it’s questioning the questioner! When I’m looking for the questions that are going to move the child forward, I’m questioning myself before I question them. “This is the question I want to ask. Where is it going to take them?” That’s the process in my brain. (Interview 2)

Cathy, Cara, and Riley were able to articulate the way they added to or revised their schema about teaching mathematics. When asked how her existing schema interacted with a new idea, Cara said, “It alters it—I add onto it and make it a bigger entity. I’m always so proud and it’s exciting for the kids when that happens!” (Interview 2). Cathy also portrayed the idea of her schema being alive by visualizing a tree: “it’s like a tree that’s got another limb grafted onto it. It’s just part of what’s there now. I

think, generally speaking, that new learning isn't planting a new tree; it's just grafting another limb onto what's already there" (Interview 2). Riley envisioned changing her thinking as "a big cyclical kind of thing" (Interview 2). Cara had a compelling visual image of the way that her students brought their unique and collaborative perspectives to the task of understanding a problem:

The picture in my mind is the question in the middle and all of the kids looking at it from different angles and coming up with different solutions depending on the angle they're looking at it. That's the way I look at bansho in my head. They're able to come in with an idea or an approach; they hear the other students and turn a little bit and see it from someone else's point of view. (Interview 2)

No matter how the participants visualized their schema, their visualizations reflected interconnectedness and growth.

One of the prime ways that all of the participants made sense of things they read, discussed, and discovered in their practice was to visualize implementation in their classrooms. This visualization happened by thinking about what a practice might look like with their students. When considering manipulatives, Laurel commented that being able to visualize how they work was an important cognitive tool: "I like to see things happening. I think I've always been a good imaginer. The concrete—although it's good—for me, the visualization is probably the stronger learning tool" (Interview 2). Cathy relied heavily on visualization as she read:

I'm doing a lot of visualization while I'm reading stuff or while I'm observing others. I try to visualize how my kids would respond to that. A lot of what goes into a successful lesson is being able to visualize before you ever start—have a

concept of “Are they going to eat this up? How are they going to approach this? What are they going to do with it?” So as I think about ideas I’m going to use in my teaching, there’s some visualizations there. I think of some of my kids—“Will it be too easy? Will it overwhelm them? If it does overwhelm them, what do I have as an alternative for that student? Is there something for the masters? How am I going to modify it for that kid?” That’s where my head is. (Interview 2)

By forming and re-forming schemata and visualizations, the participants were actively involved in making sense of their experiences. They integrated what they were learning about their students and mathematics-for-teaching into dynamic images that incorporated new learning and ongoing reflections about that learning.

Ownership

All of the teachers expressed a strong sense of ownership over their own professional learning; a sense of ownership was a condition that allowed them to engage positively in their learning about mathematics-for-teaching. Laurel made the following statement:

The way the SUM program has been created has been so important because it’s given me an opportunity to be in charge of my learning ...[to] struggle and make our way through the research with the guidance of the math team members. ...The way it’s been set up is not a coming down from the top; it’s really given me ownership of what I’m doing. Ownership is the best word because I feel like I’m in control of my own professional development. It’s a powerful thing. (Interview 1)

Their sense of ownership affected both the way that the participants looked at learning mathematics-for-teaching and the way that they felt about the experience of learning as professionals.

Within each SUM group, teachers were given the right to collaboratively choose the topic of study and had loose parameters around the way in which the collaborative study should unfold. Elaine commented, “You trust us to think on our own, throwing out some questions to get us thinking more or to get us on track” (Interview 2). The teachers sometimes investigated things on their own, as Elaine did:

I know in my learning, I’m investigating things because I’m questioning more. I want to be ready for when the kids start doing all these neat, nifty things that they’re doing. I have to know math a little better to be able to pick out strategies and understand why they’re doing what they’re doing. (Interview 1)

Elaine went on to say that this freed her to choose her own focus and to make her own connections to ideas about teaching mathematics in ways that made sense to her:

Your schema and your past knowledge of math will come out because of something they’ve said about what [the students have] done and why they’ve done it. So then you make the connections for them: “Oh, that looks like— doesn’t that look like —?” It’s reinforcing your *own* learning. And you’re seeing strategies [as they] come up with them. (Interview 1)

Laurel commented on being able to choose the pace of her learning: “You can go as quickly as you want or you can go as slowly as you want” (Interview 1).

The co-teaching aspect of the teachers’ work also enabled them to make connections to their own students. As the teachers worked together, exchanging ideas about how to connect the work their students were doing with the lesson objectives, they made connections to their own practice. They visualized doing the lesson with their own

students. Cara commented that after having co-taught a bansho in another teacher's classroom, "I want to go back into my own classroom and give it a shot" (Interview 2).

The participants all commented on the theme of ownership as playing a significant role in their students' learning. This affected the teachers' learning because it interplayed with their understanding of the role ownership played in the success of bansho as an instructional approach. Leigh stated her thinking in the following way:

It was all that talk and looking at each other and taking ownership of—it wasn't me being in charge of the group of them. ... Watching them take ownership of the solution and explain it to one another validates for me that I'm right about what I think they should be doing. ... Then I know that he or she understands what he or she is doing. (Interview 1)

The sense of ownership of their professional learning had a positive affective impact on all of the participants. They used words and expressions like motivated, trusted, energized, honoured, excited, willing, and hungry for more to describe the way they felt about their learning or the anticipation of putting that learning into practice with their students. Cara said,

I really want to try to duplicate what I saw that was valuable with my own class. I've thought that a number of times and haven't really said that, but I'm really so excited to get back and try it on my own and let my kids have that experience that I saw someone else's children have. We talked about that in SUM—not formally— but now I feel "My students deserve that. They deserve to have that." Now that I know the difference too (laugh), my students deserve to have that questioning and those experiences and connections made with them. (Interview 2)

Laurel expressed her desire to continue to learn: “It’s very motivating and leaves you wanting more. All of this experience hasn’t satisfied my appetite for my own professional learning; it’s left me more hungry. ‘That tasted mighty good and I need more!’” (Interview 2).

Having expressed their satisfaction with the mode of professional learning, they also described the difference in their thinking between when they were bored in professional learning situations and when they were satisfied. Leigh said that “you’re more excited about it; you’re more open to trying new things”; when she was not satisfied, she said she was bored: “You *don’t* think about it” (Interview 2). Riley said, “When it’s forced upon me, I get irritated. I don’t see the transfer of knowledge. I shut right down” (Interview 2). However, Riley was animated as she talked about professional learning situations about which she felt satisfaction:

You’re excited! Then it leads to you want to try it. If it doesn’t work, you want to work with it to see what would work. That kind of joy coming through teaching is a very unique feeling for people to experience in their career. Some people never experience that. You can just tell by the glow on their face—you want what they have. You want to know! It’s the collaboration, the community of learners—it’s all that that makes you satisfied and engaged. (Interview 2)

Ownership of the focus of professional learning was an important condition that enabled the participants to make connections to their own problems of practice. In addition, they observed a similar phenomenon occurring as they gave choice and ownership to their students. These two convergent understandings enabled the

participants to develop further meaning about their knowledge of mathematics-for-teaching.

Community

The final condition necessary for effective learning and development of meaning for the participants was that they be engaged with a trusted community of learners. The community provided a safe haven within which to conduct their collaborative work and from which to eventually extend their influence to draw others into their work. As they considered sharing their learning with others, they had tensions around the legitimization of their work by other communities; this was part of their ongoing struggle as learners.

Both the most frequent and the most compelling data addressed the issue of safety within the learning group and the learning network (i.e., the SUM group and initiative). Four of the participants spoke directly to this issue and referred back to it repeatedly as the prime condition upon which they believed their learning was grounded while 2 of the participants showed significantly fewer references to this condition. These 2 participants, Leigh and Cathy, had also been involved in the SUM initiative for 1.5 years less than the 4 other participants. The teachers' comments focused around two themes: characteristics of the safe community and the learning actions and thinking in which feelings of safety within the community allowed them to engage.

All of the participants considered their groups to be safe and described some of the characteristics of their safe communities. Elaine felt that the ambiance in her group was comfortable and free from fear of being laughed at, shamed, embarrassed, or made to feel less competent. She described the group as being collegial and situated her membership in it as being "a place where she did not feel she was being preached at"

(Interview 1). Riley and Laurel both described their group as being free of judgment (Interview 1).

This safe environment produced qualities of friendship. Laurel felt the support of the open, respectful relationships she had with the members of her group (Interview 1). Elaine also referred to the support of her group, enjoying the storytelling and sharing of puzzling moments in frank discussions (Interview 1). Cara keenly felt the support and encouragement of open dialogue:

I'm so grateful to be a part of it, *truly*. And to have something that I'm so excited to *be* a part of and look *forward* to the times when I get to come and be in a very safe, very intellectually stimulating group of, in our case, all women. I feel grateful for that often. (Interview 1)

Cara, Laurel, Elaine, and Riley felt a sense of togetherness and mutuality that Elaine said came from "coming through this together for so long" (Interview 1). Riley claimed that the members of her group had a friendship involving mutual respect in an atmosphere in which they were free to question and help each other (Interview 1). The connection among group members was likened to that of family by Laurel:

I spoke earlier about how I feel connected with these folks and the feeling that we're in this together. I feel they're a professional family. I think that as we go through learning about bansho and what makes it effective or how can we be more effective when we do bansho in our own classrooms, all of that conversation is all hooked in and it helps to develop the relationships that I've been talking about. (Interview 1)

The participants were asked in the second interview to describe the kinds of things about which they thought when they felt safe and to compare their thinking to the kinds of things about which they thought when they felt unsafe. This proved to be the most difficult question for the teachers to answer, often requiring probing questions and/or a reframing of the question. The participants described a process in which they felt the freedom to discuss ideas, questions, frustrations, wonderings, and “off the wall” ideas they had when they perceived the environment to be safe. When they felt safe, the participants spoke of being able to take risks, as Cathy recounted: “When I feel safe, I’ll ask what I think might be a dumb question. I’m a lot more transparent—if the whole room feels safe, we’re probably all going to be transparent and then we can all learn from each other” (Interview 2).

Cara spoke about feelings of efficacy that she derived from participation in the community. She asserted that “being a part of this group has influenced my confidence in my teaching across every part of my career, every part of my professional day” (Interview 1). Laurel and Riley also articulated that the experience with the groups in which they belonged contributed to the level of confidence they felt. Leigh commented that she felt edified by being validated by her group, saying that the group helped her to persevere when she felt challenged professionally (Interview 1).

The participants described a feeling of ease that allowed them to move from idea to idea in safe professional conversations in their groups. Cathy mentioned that everyone was engaged in the discussion, and Laurel observed that she was willing to work together to collaborate and connect with other teachers, and was “willing to pick the brains” of those around her (Interview 2). Riley and Leigh spoke of building on each others’

thoughts, ideas, and understandings that were focused on making the mathematics learning the best for their students (Interview 2). Leigh described a flow of creative thought that she experienced in her group:

There's more flow to your thoughts. ...There's more excitement. You're building on what each person has said. You might be talking at the same time. Your thinking is much more flexible and it's just freer. It's more open. And you're not worrying about filtering yourself; anything comes out. It's freeing. (Interview 2)

In contrast to the flow of thought that occurred when the participants felt safe, the teachers' thinking when they felt unsafe was consumed with thoughts about how they were being perceived by others. Every participant described being preoccupied with worries about how they would be judged by those around them. Laurel echoed the feelings of the participants:

When I don't feel safe, I tend to be a lot quieter. I don't want to be criticised by others. I'm thinking about not looking stupid. ... I'm always very careful about what I say when I'm out in the [larger school] community. ... When I'm not safe, I'm feeling hesitant and fearful and I'm much quieter. I always have the desire to participate, but I tend not to as much when I think that perhaps there will be someone who will be critical of me. (Interview 2)

Riley said that she totally withdrew in unsafe situations:

Before I say something, I will have read all the body language, I've listened to the conversation and tone of voice and decide [whether] I feel okay to say this. I analyse everyone first and then I'll say what I need to say. If I feel it's not safe, I will totally withdraw. If I feel a hidden agenda at the table I won't say anything

because I know that no matter what they say, there are no open ideas. They've already decided what we're going to talk about and it's very prescribed.

(Interview 2)

Leigh described feeling attacked in some group situations:

When I'm not feeling safe, I'm thinking about what the other people are thinking about what I'm going to say. It's not about sharing the idea that I might have, but it's more worrying maybe they're not interested in hearing this, so I'm not going to say it. Or maybe it will get shot down. Teaching is so intensely personal. ...

When something is intensely personal like that, you do get an insecurity. You feel like you're being attacked if your idea is different from someone else's, particularly if you have three or four people who think a certain thing and you're thinking, "Oh, I don't think that; I think this." And if they're particularly strong personalities—I'm someone who needs to know that [safety] for sharing or I just don't. I don't like to put myself out there just to get shot down. (Interview 2)

When they found themselves in professional learning conditions that they perceived to be unsafe, the participants only described thinking about the way they were being perceived by others.

The goal for the teachers in the SUM initiative was to develop professional capacity and generate interest and excitement about mathematics-for-teaching that would begin to involve other teachers in their schools. Teachers in the initiative were encouraged and supported to share their learning with others in their school communities when and in ways that made sense to them and to their individual school cultures.

The participants had multiple strategies for sharing their learning in their schools. All of the participants found natural opportunities to share in conversations with their colleagues. Laurel spoke of “looking for openings” when teachers in her school had heard about bansho in the system and spoke to her because they were curious (Interview 1). Elaine, Leigh, Cathy, and Laurel posted their banshos in their classrooms and in public areas of the school; Elaine had a conversation around bansho with the custodian of the school who was curious about the bansho she saw in the hallway (Interview 1). Cara and Cathy both commented that having another SUM teacher in the school contributed to the ability to have significant discussions around math at the school. Cathy said, “I think that having two of us who are enthusiastic—there’s two SUM teachers—helped in the primary division” (Interview 2). Cara conveyed her feelings about the effect of having another SUM teacher to work with her in her school community:

Having another SUM teacher at the school is making a huge difference too. ...Her attitude toward teaching math is slowly changing as well. ...When I’m listening to her, I’m thinking, “I can’t wait till those [students] make it to [my] grade” because they’re going to come in with a huge knowledge base. (Interview 1)

Laurel spoke passionately about the burden she felt to share with her colleagues what she had learned about the impact of bansho on student learning:

*I think it’s a good strategy. If I think it’s high yield, why not share? It’s a good thing. I see frustration when teachers are trying to get something, “Oh, why don’t they ever get this?” I’m thinking, “I’ve got a secret!” You just want to *share* that! You want to make it easier for them. You want *them* to see what can happen with the children’s learning. (Interview 2)*

In spite of this high level of motivation to share their learning with others, the participants experienced tensions in the process of having their work legitimated beyond the SUM initiative. On the one hand, all of the participants referred to some levels of legitimacy occurring in the system. Cara gave the following opinion:

It's certainly being recognized. You can hear principals say, "I have a SUM teacher at my school," or "We'd like to hire a SUM teacher," or "That person's a SUM teacher." And I hear my own principal say [being a SUM teacher] is wonderful. When we were interviewing for another teacher, she introduced me as a SUM teacher. It's sort of becoming bragging rights. (Interview 2)

Laurel claimed that there were some beginning signs of legitimation: "There is a certain amount of legitimacy about what I'm doing because of the presence of the math team in my room. ... All of that helps legitimize what I'm doing. However, it's still—People are paying a *little* attention" (Interview 2). Although there were these initial signs of legitimation, the participants identified four barriers to the ongoing work of capacity building in mathematics education.

The first and largest barrier the participants unanimously identified was that principals, superintendents, the board, and the province were focused on another initiative. When asked whether mathematics teaching and learning was being legitimized at the board level, Elaine acknowledged frustration and said, "It's the literacy bulldozer. It's the giant. ... We should try focusing on numeracy. I don't think literacy would drop off" (Interview 2). Laurel and Elaine recounted that their efforts to share their learning were repeatedly moved off their staff meeting agendas because items from the other initiative had gone longer than planned (Interview 2). Cathy spoke of being required by

her school administration to focus on another initiative in spite of having data that indicated there were a very few students in Grade 5 that scored better in mathematics than the students in Grade 1 (Interview 2).

The teachers articulated some thoughts about making connections to literacy initiatives in the board. All of the participants were of the opinion that there should be stronger connections between literacy and mathematics in administrators' and board personnel's thinking. However, Cara thought that mathematics was much different in its instructional approach and that approaches to literacy could not be adopted for math (Interview 2). Riley was of the opinion that literacy could be approached through bansho, with teachers making connections across pieces of student work in bansho style (Interview 2). Leigh felt that the work of another initiative seemed to be separated into separate silos of knowledge by the funding formula and focus of the board and its administrators:

I don't think they realize the mixed messages they give. They don't practice what they preach. When you present math stuff, you draw attention to the aspects of literacy that are involved in the math. I never walk away thinking that literacy is not a part of math. But when you're sitting in literacy, they tell you that you have to integrate, integrate, integrate. But they don't model it. Instead of it being literacy here and then you should be integrating science or social studies or whatever, it should be literacy is in all of this. It's the underpinning of it. ... If you need to take time for direct instruction, of course you do it, but it's all part of it. (Interview 2)

A third barrier identified by Cara, Laurel, and Leigh was not having time to collaborate (Interview 2). Although the other three teachers did not specifically mention this as a barrier, they highly valued the time they were given to collaborate and saw collaboration as critical to their learning mathematics-for-teaching. All of the participants were given the opportunity to describe what they would do to teach another teacher effectively about bansho. Laurel mused about what she would do if she had optimum time to work with her colleagues:

I would actually *invite* someone in. They can be at what their comfort level is; watch me go through the process, watch what it looks like, what it sounds like. ... *Then* [I would] go back and maybe give a little background and show and talk about why I did what I did, have them question me. Then the next step would be to offer to do one with their class or to plan one with them and have them go off and then come back and do a debrief—what worked, what didn't. (Interview 1)

The fourth barrier that three of the teachers identified was apathy. Cara linked a sense of apathy to not being given time during school hours to do collaborative work in mathematics yet being given time to study literacy (Interview 2). Laurel linked time, funding, and the desire to please a principal whose focus was not mathematics to the feelings of apathy toward math teaching and learning (Interview 2). Cathy spoke passionately when asked about whether she thought her work was being legitimated in her school. When prompted to say what needed to happen to make changes to the apathy she saw, Cathy put her theory into words:

[What needs to change is] the attitude that where these kids are coming from, they can't learn. When I started, at the very first staff meeting [someone said], "We don't take education too seriously here because there's no point." That was said in front of the entire staff. It was directed to me, but the entire staff was like, "Yeah—obviously." That kind of a mindset takes a while to flush out, and we're not there yet. ... It's changing. We're getting there, but it's not something that's happening overnight. It's going to be a long process. It's a process of people literally changing, like changing staff, and for some it's a process of growing—but for some—Somehow you've got to get that core thinking changed, that [says] "Yes, these children can learn." Then from there, you've got to get the thinking changed that says "Yes, there are other ways [of teaching] that may even be superior to what I'm doing." And then you've also got to get the thinking changed that says "This is where I want to put my energy." So there's a three-pronged plug here. Number 1, the kids are worth it. It's worth giving up my time for the kids. Number 2, there *are* better ways than the ways I do things. What I learned in university—there's more than that. That [teaching] is evolving. Then you've also got to have them to the point where they say, "This is where I want to put my energy." (Interview 2)

The community in which they collaborated and practiced was the major condition that supported the teachers' learning. Providing a safe arena for the flow of ideas concerning practice to emerge also provided the safety necessary for the participants to take instructional risks and to elicit feedback from one another in order to be more precise in their instruction. As they discussed ideas, they made connections to other areas

of professional learning and deepened their understanding of classroom, school, and board cultures.

Chapter Summary

The framework that emerged from the analysis of the interview data consisted of three processes and two conditions that led to the participants' professional learning processes. Informing with resources, engaging with students, and visualizing and schematizing were the three key processes that emerged; ownership and community were the necessary conditions that facilitated the participants' learning and development of meaning in their professional learning. The following chapter will offer a discussion of the findings as they connect to the literature.

CHAPTER FIVE: DISCUSSION AND IMPLICATIONS

This study set out to examine how teachers learn about mathematics-for-teaching by engaging in the use of Ontario bansho as an instructional strategy within their communities of practice. Embedded in a constructivist approach to teaching mathematics, Ontario bansho is a means through which teachers use the thinking revealed in students' problem solving solutions to construct interconnected pathways of conceptual development of the big ideas of a topic in mathematics. Ontario bansho was learned and practiced by the 6 participants in the study within a network of professional learning communities, whose members supported one another through collaboration to co-plan, co-teach, and reflect on mathematics lessons. The study informs the body of professional learning literature in that it examines processes and conditions in which teachers learned about the complex interweaving of content and pedagogy in research-informed ways that have transformed their practice.

Following an interpretive phenomenological approach, the study employed a qualitative research design in order to examine and to make visible the ways that individual teachers navigated through the phenomenon of teaching using Ontario bansho within their communities of practice and thereby added to their knowledge of mathematics-for-teaching. The use of this approach also allowed the social aspect of the participants' knowledge construction to emerge. This chapter will include a brief summary of the study, a discussion of the major contributions of the study in relation to the literature, the implications of the study for both theory and practice, and a brief discussion of my personal learning as a result of completing this research.

Summary of the Study

My interest in conducting this study arose from my role as a professional learning facilitator working with teachers in mathematics teaching and learning. Kubota-Zarivnij (2007), a provincial professional learning facilitator, made the claim that Ontario bansho enabled teachers to learn more about mathematics-for-teaching. My own learning and implementation of Ontario bansho within my communities of practice had contributed significantly to my professional learning of mathematics-for-teaching; I was curious to know how teachers' professional learning occurred within their communities of practice. From this curiosity emerged the central research question: How does teachers' implementation of Ontario bansho within their communities of practice inform their professional learning process concerning mathematics-for-teaching? The following two key questions also guided the study: (a) What processes support teachers' professional learning of content-for-teaching? (b) What conditions support teachers' professional learning of content-for-teaching?

In considering how to investigate these questions, I discovered Kvale's (1996) comment about phenomenology: "Phenomenology attempts ... to make the invisible visible." Smith (2008) states that "phenomenology leads from conscious experience into conditions that help to give experience its intentionality" (p.1). These two quotations resonated with my interest in examining teachers' thoughts in order to understand the factors that contributed to make collaborative learning of Ontario bansho transformative. The choice of a purposive sampling of teachers as participants arose from both convenience and from my work with a large group of teachers who I knew were members of a community of learners. Conducting interviews and following an interpretive

approach to data analysis allowed me to study the ways in which teachers construct meaning and create interpretations through their daily social interactions, and enabled me to construct a model of professional learning. The emergent model was made up of three processes in which the participants engaged and two conditions that supported the three processes. The three processes occur in a nonpredictable sequence and may occur both simultaneously and in recurring ways.

The first process, *informing with resources*, included many different external means that teachers used to add to their knowledge and understanding of content. These incorporated professional learning books, research articles, traditional workshops, teaching materials, professional learning videos, and so on. This also included the kind of informing that goes on when speaking or observing a knowledgeable other using an instructional strategy.

Engaging with students was a process of putting theory into practice in co-teaching or coaching situations in classrooms with students. Engaging in this theory-to-practice work together allowed the teachers to share an experience about which they could later converse. The experience also allowed teachers to focus on student learning through direct observation of students at work with content.

In the third process, *visualizing and schematizing*, the participants engaged in forming and re-forming schemata about both the teaching of mathematics and mathematics-for-teaching. As they participated in collaborative experiences with their students, the teachers reported making new connections to the ways in which the manipulatives, models, big ideas, student strategies, and pedagogical strategies were interdependent. The participants also developed meaning as they visualized themselves

engaging in teaching their own students while they watched someone else teach and while they read professionally. This visualization process was instrumental to the teachers' forming an enhanced sense of personal capacity and efficacy.

Ownership of the direction of the professional learning was one of two conditions that supported the three processes. The teachers were highly engaged in their professional learning process because the structure of the professional learning experience allowed for and was driven by questions that arose from the learning groups' own practices. Ontario bansho provided a structure for classroom instruction that allowed students to experience similar levels of ownership and engagement. Subsequently, the teachers reported that the students experienced higher levels of efficacy; because their students were experiencing success, the teachers also spoke of feeling higher levels of agency and self-efficacy. This enhancement proved exciting and prompted the teachers to engage in more collaborative learning opportunities, establishing a culture of learning among their professional learning groups.

The foundational condition for the participants' construction of knowledge of mathematics-for-teaching was *community*. Active engagement in a safe professional learning group supported their learning and allowed them to take risks, to ask questions of and reflect on their assumptions and practices, and to engage in a culture of inquiry. *Not* feeling safe in other contrived groupings had turned their focus away from learning toward self-preservation and face-saving. The perceived safety of the professional learning community also allowed excitement about the participants' learning, about the positive impact on student achievement the changes in their practices were making, and about their increased sense of efficacy to contribute in a positive way to the shared

culture of their professional learning groups. When they felt support from school and district leadership communities, these positive effects were further enhanced.

Discussion

This study contributes to the body of knowledge concerning professional learning. The discussion of results will be organized around five topics of particular interest. The discussion begins by addressing the central question concerning Ontario bansho and its potential to impact the professional learning of mathematics-for-teaching. The remaining four topics address unanticipated outcomes of the research.

Ontario Bansho: Professional Learning of Mathematics-for-Teaching

Using Ontario bansho as an instructional strategy within a learning community was a powerful way for teachers to learn about many aspects of mathematics-for-teaching. The study confirmed Boerst et al.'s (2008) assertion that developing knowledge of mathematics-for-teaching is a complex process. Although this was an anticipated outcome of the study, the variety of ways that the participants constructed this knowledge was surprising. The teachers made strong connections to the research about which they read and organized their learning into schemata that made sense to themselves. They consistently used metaphoric language that revealed that they visualized a learning-landscape of interconnections with overarching big ideas guiding and giving coherence to the instructional paths down which they went. They found that the landscape metaphor allowed them to honour the complexity of mathematics instruction and gave them a framework within which to consider the interconnected nature of learning.

The participants in this study worked to hone their questioning skills within bansho lessons as an integral part of the assessment-for-learning advocated by Earl

(2003). They used questioning in the way advocated by Anthony and Walshaw (2009), realizing that “a wrong answer might indicate unexpected thinking rather than a lack of understanding [and] a correct answer may be arrived at via faulty thinking” (p.18). In this way, the participants used questioning as a means through which they discerned how to build students’ understanding in a constructivist manner.

The bansho lesson enabled the teachers to combine information gleaned through effective questioning to make an explicit conceptual connection of strategies and thinking from one student’s work to another student’s work in the bansho, as advocated by Takahashi and Yoshida (2004). This was part of the teachers’ sense-making process. The teachers also used the bansho to annotate student work, showing students how to represent their mathematical thinking. They found that the consolidation of the big idea into an explicit statement at the end of a lesson was a powerful learning tool for their students because it connected all of the solutions and validated each student’s problem solving. In this way, the bansho served as the “collective thinkpad” described by Kubota-Zarivnij (2007).

The teachers recognized the complexity of the system of knowledge they were trying to learn, supporting Ball and Forzani’s (2007) claim that it is not just the content, learners, or the environment, but it is “understanding the dynamic relationships among them that constitute the core of the educational process” (p. 531). This study confirms that teachers need to learn content-for teaching in ways that honour the depth and complexity of the content, the pedagogy, and the interplay of the two.

Teaching mathematics using Ontario bansho also influenced the school’s culture of collaborative learning of mathematics-for-teaching. The physical artefact of the

completed bansho lesson proved to be useful in promoting spread within schools. In the ripple effect described by Flynn et al. (2009), as teachers put their banshos in public areas, other members of the school community ask for explanations, enabling the teachers to share their learning with other teachers, parents, and nonteaching school staff. In this way, using Ontario bansho as an instructional strategy contributed to both the teachers' and their students' learning, and had the potential to influence the learning culture of school communities.

Ownership as Change Catalyst

An unanticipated theme emerging from this study is that ownership forms a critical condition for teachers to become deeply engaged in making changes to their practice. This finding confirms the research of Kaser and Halbert (2008), who found that one of the key strategies to effect a change of mindset toward deep learning was putting learners in charge of their learning within respectful environments of inquiry, evidence, and choice. In my study, putting teachers' questions of practice at the centre built a sense of ownership and accountability to enact the agreed-upon actions in their classrooms. As the teachers saw increased levels of student understanding, they were convinced of the merits of persevering in learning the new instructional strategy, thus shifting beliefs about what it means to teach mathematics effectively. The respect that the facilitators of the professional learning had for the teachers in this study enabled the teachers to examine their practice openly, to take personal risks, and to be open to new ideas. They described high levels of personal involvement in the professional learning and were willing to challenge their preexisting ideas and to try new instructional strategies. This shift supports Guskey's (1986) statement that teachers will change their beliefs only when they

see positive results in their students' learning; it was the sense of ownership of their learning that the teachers saw as a prime catalyst for their shift in belief.

The participants in my study spoke about the effects of experiencing a lack of ownership of their professional learning. They spoke negatively about feeling forced to study topics about which they had little interest and had little input into the ways in which mandated collaboration in other initiatives was conducted. Hargreaves (2007) argues against leadership communities that are too directive in their objectives for teachers because teachers' engagement in learning will "accompany integrity through the exit door of professional withdrawal" (p. 37). Tschannen-Moran (2001, p. 308) describes such mandated efforts as contrived collaboration in which teachers complain that they have not been given any real influence over the outcome of decisions. Garmston (2009) advocates that leaders provide a safe way to surface tensions in order to receive genuine feedback. While the teachers in my study were compliant, they reluctantly supported their leaders who proposed such directives but lamented the fact that they felt they had no voice to give feedback to anyone who would listen. In addition, they felt that the good work they were doing in mathematics was ignored by leadership and considered less important. In other words, lack of ownership led to a culture of resentment in which little learning occurred and which was ultimately counterproductive to the professional learning goals of the initiative.

Knowledgeable Others as Resources

A third surprising element to emerge from this study was the role that knowledgeable others played in the participants' learning. Goddard et al. (2004) suggest that vicarious experience in which a skill is modeled by someone else can influence the

way that people learn, and the content-focused coaching of West and Staub (2003) hinges on teachers working in pairs in which one teacher mentors another. Although the facilitators of the professional learning groups were mathematics-for-teaching content specialists, they placed themselves as equal members of the learning community and were uncomfortable with taking a role that elevated their expertise as a model for good practice. The participants were appreciative of this style of leadership; however, they all spoke enthusiastically about the way they benefited from the opportunity to watch the facilitator as *knowledgeable other* conduct a bansho lesson. The experience enabled the participants and their professional learning groups to see how the lesson was connected to organizing big ideas and how questioning might be enacted, and it permitted them to visualize themselves enacting a similar lesson with their students. Engaging in dialogue with group members about the instructional decisions made by the knowledgeable other allowed the teachers to connect the theory with practice in a way that empowered and excited them to try it themselves. In this way, this study builds upon and connects the body of knowledge about instructional coaching and professional learning communities.

Engaging With Students as a Way of Knowing

Much of the work that the participants' professional learning groups were involved in was geared toward doing action research (Lieberman, 1996) and lesson study (Stigler & Hiebert, 1999). It was surprising that the teachers learned about teaching mathematics by uncovering and building on students' *actual* understandings as well as by studying lesson construction. The vast majority of professional learning opportunities occur without the presence of students in the actual professional learning sessions.

Enacting lessons with students allowed the teachers to see for themselves the nuances and fragility of students' understanding as students built their understanding of the mathematics concepts being discussed in the classroom. Practicing together with the students allowed the teachers to focus on different aspects of the lesson during their observations of students at work; Curry and Killion (2009) describe this as both the micro-level and the macro-level of professional learning. Micro-level learning might include developing and understanding of the way using one mathematical model over another allowed a student to better understand a concept. Macro-level learning might include understanding why making an explicit connection to a big idea is important in all mathematics lessons. The discussion that ensued allowed for multiple perspectives when talking about these different aspects of lessons. The richness of this experience and dialogue, and the extent of the teachers' learning, contributed to a sense of excitement and motivation that all of the teachers in the study reported. For the teachers in this study, engaging with students was a different and highly useful way of knowing.

Joyce and Showers (1996) found that in-classroom assistance in the form of peer observation and coaching is critical to the transfer of more complex teaching skills. West and Staub (2003) suggest using one-on-one peer relationships to hone mathematics-for-teaching instructional strategies while engaging in the act of teaching. Boerst et al. (2008) advocate that pre-service teachers need to learn about the intricacies of teaching by basing teacher education on the *work of teaching* (p. 5). This study embeds the collaborative work firmly in the classroom in order to learn from the perspective of actual practice. Although engaging with students is not uncommon in the professional learning

literature, this study found that the collaborative work done with students played a highly significant role in contributing to the teachers' construction of meaning.

Enjoyable Professional Learning That Builds Efficacy

The most unexpected outcome from this study was the sense of enjoyment and satisfaction that the participants brought to the descriptions of their collaborative mathematics professional learning group experiences. Their learning gave them an ongoing flow of ideas. They described being fascinated by their students' learning and by the connections they were making in their thinking about mathematics-for-teaching. These experiences contributed to what Csikszentmihalyi (1990, pp. 58-61) described as *flow*, a state in which heightened focus precludes distracting thoughts and which produces a strong sense of enjoyment. This study also confirms Mitchell and Sackney's (2009) claims that when a balance of respect and ownership exists, teachers will be excited and energetic about their learning.

Having a sense of ownership allowed the teachers in this study to attribute the increases in student achievement that they saw in their classrooms to the results of their own efforts. Mitchell and Sackney (2009) state, "if [learners'] different dispositions and interests are honoured, and if they have opportunities to feel successful, they are likely to take risks, try, and persevere" (p. 151). There are also strong connections to the work of Bruce and Ross (2008), who found that teachers' sense of efficacy is connected to student achievement as they come to believe that it is their actions that produce positive academic outcomes. Although the teachers in my study reported that their students were better able to engage mathematically in the problem solving bansho lessons, this study did not attempt to make correlations to student achievement.

Lieberman (1996) advocates for collaborative work to be embedded in a culture of inquiry. Mitchell and Sackney (2000) embed effective and productive dialogue within the professional learning communities by advocating for openness to divergent thinking and opinions as part of a discourse community whose function is to closely examine practice, beliefs, and attitudes as they consider novel approaches to problems of practice. The participants in this study had such communities of practice and enthusiastically described the merits of these communities with respect to the way they enabled them to move forward in their learning. Their communities had a climate of trust that Flynn et al. (2009) claim is essential to “give teachers the freedom not only to take risks and experiment in their classrooms, but to do so in front of their colleagues. This trust also gives them the space to self-reflect openly on their teaching practice” (p. 11). In fact, the teachers themselves said that they could not have made the progress they had made without this kind of support from their communities. Their positive interdependence built a sense of efficacy that was both individual and collective. Their successes fed back into the efficacy-building cycle.

As striking as the positive effects of the participants’ mathematics professional learning community were, so was their reaction to converse situations. The teachers in this study unanimously stated that the absence of trust and safety in a learning community led their thinking in very unproductive directions. Not only were they *not* thinking about the content with which they were supposed to be engaged, they were completely distracted by thoughts of self-preservation. None of the teachers could describe anything that they learned when they felt unsafe in a group that was supposed to be learning together. Instead of feeling excitement, they felt resentment.

Implications for Practice

Improving what teachers know and are able to enact in the classroom is the goal of professional learning because teacher knowledge about content-for-teaching is the single largest predictor of student achievement (Ball et al., 2005). This study has confirmed the level of complexity inherent in teachers' learning of mathematics-for-teaching. It has also elucidated a model of professional learning that is useful for the design of professional learning in all content areas. Facilitators of professional learning can use the processes and conditions uncovered in this study to design professional learning initiatives to make their work more intentional.

This study confirms that facilitators of professional learning should consider a wide array of experiences and materials as resources to inform the study of content-for-teaching. Not only do resources instruct, but they also serve to create a meaningful disturbance that may begin the journey to a profound shift in thinking. An unexpected resource to the teachers was watching a knowledgeable other conduct a lesson. However, a caution might be raised with respect to this; the teachers did not describe this as a modeled lesson with an agenda for observation but rather as a lesson in which they were free to observe and think about those aspects of the lesson that made sense to themselves. Because when and which kind of resource will be germane to teachers' learning cannot be predicted, one resource should not be offered for everyone to study, but rather a wide variety of resources should be offered to learning communities.

The teachers highly valued the inclusion of a knowledgeable other in their communities of practice. This knowledgeable other gave direction to the group without being prescriptive by coordinating resources, acting as a resource through the use of

effective questioning and instructional strategies, and being a source of ideas to spur deeper thinking. Bandura (1986) has postulated that vicarious experience such as that described by the participants in this study allows teachers to imagine themselves enacting similar experiences with their students and this in turn contributes to personal efficacy. Because efficacy is also correlated to student achievement (Ross, 1992, systems wanting to impact student achievement should include opportunities for teachers to collaborate with knowledgeable others.

This study points to significant learning that can happen when collaborative inquiry is situated in the classroom. As teachers put their learning into practice, they built meaning about the nuances of putting theory into practice. In addition, when lessons are co-planned and co-taught, teachers have a shared experience that can be viewed from multiple perspectives, taking into account the student learning that occurred. This is fine fodder for the reflective discourse advocated by Mitchell and Sackney (2000). Job-embedded professional learning, then, should not merely be situated in staff rooms looking at the work students produce, but rather be situated with students so that the full range of their thinking and the ways that they construct meaning about content can become part of the data set that teachers use to inform their instruction.

Working with students also gives teachers opportunities to practice through the sharing of in-the-moment decisions such as: “What are good questions to ask to uncover this student’s thinking?” These kinds of in-the-moment decisions may also be based in teachers’ knowledge of content-for-teaching. Building knowledge about these complex aspects of practice not only builds teachers’ sense of personal efficacy, but also contributes positively to their collective sense of efficacy that Goddard et al. (2000)

conclude are predictive of increased student achievement. Professional learning programs should integrate this process into their initiatives so that teachers can learn *in situ* about the bridges from theory to practice.

Understanding how topics within content areas are built conceptually involves the schematizing of many interconnected ideas. This study confirms that this complex work benefits from the input of resources, professional dialogue, and direct work with students. If teachers are to engage in this kind of schematizing, facilitators of professional learning must both honour and expect teachers to engage in making sense of complex landscapes of learning. In addition, teachers need opportunities to dialogue about instructional strategies and be able to visualize themselves enacting these strategies with their students (Goddard et al., 2000). They need to be able to personally create the dynamic mental models around instruction advocated by Earl (2003). Over-scaffolding, over-simplification of complex ideas, and over-reliance on others' schemata rob teachers of the opportunity to make connections and form schemata that make sense to themselves. Facilitators of professional learning do well to acknowledge learning in its complexity.

The teachers in this study valued the sense of ownership they were given over their learning within their collaborative groups. Teachers are far more likely to enact new learning in their classrooms if the stuff about which they learn is practical and useful in their teaching (Lieberman, 1996). In addition, when teachers are given a voice in their professional learning decisions, they overcome resistant inclinations (Hargreaves, 2007). The teachers in this study felt powerless to alter the decisions made by others about their professional learning; they believed there was no way to give feedback to those in authority without personal loss. In sharp contrast, when given ownership of their

learning, the teachers were excited, were energized, felt high levels of efficacy, and were eager to share their learning with their colleagues by drawing them into their classrooms and engaging in co-teaching with them. This ripple effect (Flynn et al., 2009) has the potential for changing school cultures into the kinds of excited, motivated learning communities advocated by Mitchell and Sackney (2009) and could positively influence the collective teacher efficacy that Goddard et al. (2000) found to contribute to increases in student achievement. This study links a sense of ownership to personal and collective efficacy to impact student achievement.

Care must be taken to ensure that trusting relationships are built within learning communities, recognizing that without this trust, there will be little risk-taking, little self-reflective practice, little schematizing, little building of feelings of self-efficacy, little learning—the teachers will be more concerned with self-preserving thoughts. Katz, Earl, and Ben Jaffar (2009) state teachers build trust in the community by engaging in the work of teaching. While this study confirms that stance, it also provides examples of teachers engaging in aspects of the work of teaching yet feeling unsafe and being distracted by worry about how they were presenting themselves.

In this study, the excitement about learning within four of the participants' communities of practice began to spread to their school community. This occurred among the teachers who had been involved with their professional learning group for a longer time. If leaders believe that it is from the grassroots that teacher-leadership can provide the momentum to change school cultures to be more committed to increasing student achievement, then it makes sense that school systems do what they can to empower such grassroots leadership.

System-level leadership also plays a part in the professional learning community. It is system-level leaders who must fight the pressure of getting results quickly, advocating for sustained professional learning opportunities in communities of learners with whom they share questions of practice. They must recognize that ownership of professional learning goals is critical not only for buy-in, but also for spreading effective practices within schools and systems; the authoritarian imposition of professional learning goals at best breeds compliance and at worst the poison of resistance. System leaders must recognize that content-for-teaching knowledge is critical; increased student achievement will not come from focusing on general literacy or on general learning skills because subject-specific expertise defines the instructional moves that teachers make. These leaders will ensure that there is a plan to retain and build capacity using the expertise developed within professional learning communities. Finally, system leaders must find ways to gather honest feedback, so that they may use that feedback as assessment for the facilitation of teachers' professional learning.

Implications for Theory

The model generated in this study connects various elements that contribute to the participants' learning and development of meaning. It roots meaning-making in the conditions of *ownership of inquiry* and *safe and trusted communities*. Members of the community engage in learning about teaching by engaging in the act of teaching together.

Each of the three processes described in this study supports a constructivist approach to teachers' professional learning theory. The process of *informing with resources* introduces the idea that teachers build meaning about content-for-teaching by interacting with texts and materials, and by discussing with and observing knowledgeable

others. These actions inform teachers' thinking and cause them to visualize themselves enacting the teaching moves being observed. As they do this, they simultaneously integrate new insights into their existing schema about the content and the ways that contribute to a more robust understanding of teaching and learning within and about that content. In a similar fashion, the process of *engaging with students* ensures that teachers put their growing schema about content-for-teaching into practice. As they craft lessons about content, they see in the moment how students react to and think about the content. These observations contribute to the teachers' understanding and schema about how different aspects of content-for-teaching might be enacted in the classroom to further build students' understanding. Teachers in turn may visualize the changes they might make when they next engage with this particular content. This model implies that if teachers are going to construct adaptive understanding of content-for-teaching, the site of their learning must actually be in classrooms, where they learn as they engage in the act of teaching.

However, it is not enough that the sites of teachers' professional learning change. So too must the content of their professional learning. This model implies that it is not appropriate to deliver professional development sessions that tell teachers what they will learn. Rather, teachers ought to be given the license as professionals to inquire about genuine problems of their practice as it relates to learning about content-for-teaching. This is a shift in paradigm from one of management and perceived control of the pieces of teachers' professional learning toward an acceptance of the emergence and construction of understandings that have direct application and implications for teachers'

own professional practice; teachers can be entrusted with ownership of their professional learning.

Finally, the notion of communities of practice should be seen not as a structure for managing groups of people. Instead, community should be seen as a critical underpinning of teachers' professional learning. As people work together over time to understand practices that are of vital interest to them, a sense of safety grows, which allows openness to new ideas and practices to flourish and honours the excitement and sense of self-efficacy that takes hold when people are really learning. Community supports learning and as such deserves time and attention so that teachers' learning is made the most likely outcome of the professional learning.

The three processes and the two conditions contributed to the development of meaning concerning content-for-teaching in ways that were complex, recursive, nonpredictable, and highly dependent on context. This has been captured as directionality in the visual representation in Figure 2; the directionality of the arrows should be considered as movement among the elements, and not as a lock-step plan. As learners engage in any process (and they may occur simultaneously), and as the learning is supported by the learners' ownership within the learning community, meaning is constructed.

Theories of professional learning must incorporate and honour the complexity inherent in constructing meaning about the many varied aspects of teaching. In this respect, no model can be seen as definitive. Nevertheless, robust professional learning theories provoke a look at professional learning from previously unconsidered perspectives that are useful to inform the practices of both teaching and professional learning.

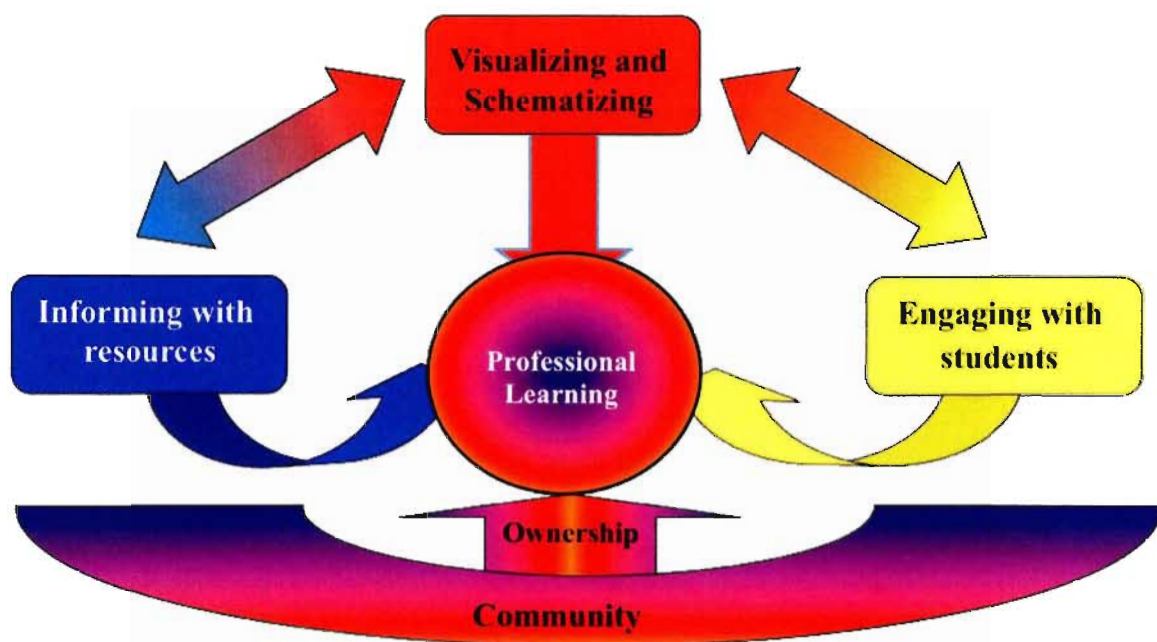


Figure 2. A model of teachers' professional learning within a community of mathematics-for-teaching practice.

Implications for Further Research

Recent research by Bruce, Esmonde, Ross, Dookie, and Beatty (in press) finds that when teachers engage in supportive, collaborative, and classroom-embedded professional learning programs, there is no ceiling of learning. Rather, there is deeper engagement in professional learning among these teachers than among those teachers who are new to professional learning strategies. Conducting a longitudinal study to examine the effects on teacher learning and student achievement of a professional learning initiative designed with the conditions and processes of the model proposed in this study in mind would be beneficial. Such a study might examine quantitative data over time, looking at knowledge of mathematics-for-teaching using Ball et al.'s (2005) CKT-M (Content Knowledge for Teaching Mathematics) scale, at building efficacy over time using Woolfolk Hoy, Hoy, and Kurz's (2008) Efficacy Scale, and at student achievement data over time. Such a study could also examine qualitative data collected from interviews or case studies with involved teachers, to uncover the long-term effects for teachers and students.

In this study, Ontario bansho was found to be an effective instructional strategy for learning about mathematics-for-teaching within communities of practice. Further research might examine the impact on teachers' learning of mathematics-for-teaching of creating professional learning communities for the sole purpose of learning how to teach Ontario bansho. This might be a mixed methods study in which qualitative data about the mathematics-for-teaching teachers learned was combined with quantitative data that could be collected using the CKT-M and Efficacy scales combined with student achievement data. Another qualitative study might request teachers to construct personal

landscapes of mathematics learning to examine the kinds of mathematical connections they made to their schema concerning a given topic.

This study surfaced ownership of professional learning content and processes as a catalyst for change. Further investigation might focus more intently on this condition. Alternatively, existing professional learning programs might be examined to discover the extent to which ownership of teachers' learning was perceived, and how ownership or a lack thereof enhanced or detracted from professional learning goals. Such a study might give practical feedback for improvement to stakeholders.

Further research might also be conducted to learn more about the role of knowledgeable others in professional learning. A comparative study might be conducted in which learning communities that were not facilitated by a knowledgeable other are compared and contrasted with those that were facilitated by a knowledgeable other. Alternatively, a study might be undertaken that interviews knowledgeable others to better understand how teachers become knowledgeable others, and how and to what extent their expertise is used by boards of education. Measures of content knowledge such as the CKT-M scale across subsections of teachers could be a powerful cross-check for such a study.

The way that engaging in working together with students informed teachers' learning exposes another area for further research. Once again, a study that contrasts professional learning programs that situate collaborative professional learning in co-planning and co-teaching with initiatives that involve one-on-one coaching and/or with those that look at student work but which do not involve actual co-teaching would be beneficial. Such a study might provide further evidence to inform decisions about

moving job-embedded professional learning away from merely being site-based (i.e., occurring in schools) toward being classroom-embedded. Such a study might also confirm the benefits to teachers' learning of communities of practice sharing actual teaching experiences.

Finally, further research into the roles that teacher efficacy and excitement about professional learning play in shifting teachers' practices would be beneficial. Understanding these critical roles better would draw attention to the importance of building underlying structures that underpin the building of trust and other affective considerations that support learning within professional learning communities.

Personal Learning

The purpose of engaging in research at the master's level is to develop personal capacity for educational research. As such, research is the content about which I have been learning. By applying the model I have proposed in this study, I see that the model is powerful enough to describe the processes and conditions that have contributed to my learning about educational research. Therefore, I will use this as a framework to report on my personal learning.

I have been able to inform myself with resources, including the many studies and pieces of literature I reviewed. Participating in a community of graduate students gathered under the advisorship of Dr. Coral Mitchell allowed me to hear others further along in the process describe their learning about researching. I was able to engage in discourse with Coral as the knowledgeable other and with my peers about the various aspects of research prior to engaging in those aspects of the process myself. This

experience proved to be invaluable as I continually engaged in negotiating meaning about engaging in research.

My practice consists of working with teachers to facilitate professional learning of mathematics-for-teaching, but it was not until I engaged with teachers in interviews about their practices that I was able to have first-hand knowledge of the ways in which they understood the process of meaning-making with respect to their problems of practice. Thinking about the participants' thinking gave me new insights into the ways teachers negotiate meaning about mathematics teaching and learning. I learned in an entirely different way when I engaged in dialogue with them about their practices enacting Ontario bansho within their communities of practice.

Attempting to incorporate the many theories about learning from within the disciplines of mathematics education and professional learning, together with the data gleaned from interviews with the teachers, put my ability to schematize to the largest test of my lifetime. Making connections between theory and practice was a daunting task, facilitated by my growing schema for conducting qualitative research. In turn, enacting the data collection, data analysis, and report writing phases of educational research helped me to develop new meaning about why each part needs to be approached from the viewpoint of a researcher. To support me along the way was our graduate study community through whom I was able to visualize myself accomplishing each of the phases of research.

Perhaps the largest surprise to me as I learned about educational research was the enjoyment that I experienced as I constructed my own understandings in ways that made sense to me. I have expanded my own sense of efficacy; I am fully capable of conducting

educational research. That my knowledge construction has occurred in an area of research that impacts my professional work as a facilitator of professional learning makes my apprenticeship to research processes that much more valuable; I can, and have, used my own theory to inform my own practice.

I have benefited most from being able to work in so many communities of learning as I build personal meaning about the professional learning of mathematics-for-teaching. My graduate study group, facilitated by Dr. Mitchell, supported my immediate research. The participants with whom I have worked have informed my thinking, and our discussions have illuminated our collective understanding of mathematics teaching and learning. I have benefited from countless discussions with professional learning facilitation teams about mathematics and other content areas at the district and provincial level. All of these communities have contributed to my professional learning and development of meaning about the role and enactment of educational research. Community is a critical condition for learning.

Finally, I have also had personal learning about the need for theories-of-action to be developed and disseminated to other learning communities. These theories inform us, challenge us to re-schematize, and drive us to improve. Students matter. Helping them to learn and to navigate successfully is our moral imperative. They are our future. As Mitchell and Sackney (2009) state:

Although the lesson of contextually constructed pathways might seem daunting, it is also one of the most exciting and energizing lessons of sustainability. It frees people to notice compelling disturbances and to build meaningful responses that can, in fact and not just in theory, make a deep and lasting difference in the lives

of their students and colleagues—and thus in their own lives. This lesson is not only respectful and enabling, but it is also deeply ethical. (p. 195)

Final Thoughts

Managing complexity in professional learning is not possible any more than it is possible to *manage* an ecosystem. However, leaders *can* pay attention to processes and conditions that make professional learning more likely. The model of professional learning put forth in this study is one way to conceive of being intentional about the elements that contribute to teachers' building of meaning. By paying attention to the processes of informing with resources, engaging with students, and visualizing and schematizing, as well as to the conditions of ownership and community, leaders will also contribute to teachers' personal and collective efficacy. Paying attention to these processes and conditions will increase the likelihood that gains in student achievement will occur. This model provides a framework for facilitators and leaders to plan for effective, content-relevant professional learning by placing teachers, students, and their learning at the heart of professional learning.

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Appendix A

Interview Guide for First Interview

Interviewer:

- Before we begin, I would like to remind you that you may choose not to answer any question and that you may withdraw from the study at any time with no penalty.
- As part of the ethics protocol, I must refer to each of the participants and all their responses by a pseudonym or fake name. By what pseudonym (a name other than your own) would you like to be referred?

Introducing questions

- How did you first learn about Ontario bansho? (uncover existing communities of practice)
- Can you tell me about a time when you used Ontario bansho with a problem your students solved? (activate prior knowledge)

Direct questions

- Was there a moment in your mathematics teaching career that you made a fundamental change in the way you teach mathematics? What do you think caused this change in your thinking to occur?
- How would you describe your mathematics teaching community of practice? (characteristics of community of practice)
- Please describe any other groups you might consider to be another of your communities of mathematics practice? (is there a network or interconnection of communities?)

- In what ways has learning about Ontario bansho **in a professional group of teachers** contributed to your understanding of what it is to teach mathematics effectively? (perceived effect of learning in a social context)
- Learning about and teaching through Ontario bansho has been described as a social process. Do you agree with this?
 - If no, go to the next question.
 - If yes, ask: Describe how the mathematics community supports your learning.
 - How does the community support the learning of your students?
(perceived effect of learning in a social context)
- Has your involvement with your community of mathematics teachers contributed to your understanding of pedagogy? In what ways? (learning of pedagogy)
- Could you describe some of the things doing Ontario bansho as a learner or a teacher has allowed you to learn? (mathematics-for-teaching learning)
- Has doing Ontario bansho as a learner or a teacher helped you to understand something in a deeper way? Has it consolidated any learning you may have already done? (mathematics-for-teaching learning)
- Have you ever learned something about mathematics as you were doing a bansho lesson with your students?
 - If yes, ask: Please describe what you learned. (mathematics-for-teaching learning)
- Do you remember telling anyone about what you learned?

- If yes, ask: Who did you tell? Why do you think you told them?
(motivation for social engagement about mathematics learning)
- Do you think learning about or doing Ontario bansho helps to make connections?
(mathematics-for-teaching)
 - If no, go to next question.
 - If yes, ask: With what do you think Ontario bansho helps to make connections?
 - With what do you think Ontario bansho helps to make mathematical connections?
- What aspect of doing Ontario bansho do you value most and why? (what is the most important element in the emergent framework – what is/are the driver(s)?)
- In what ways does doing bansho contribute to your classroom or school culture? (teachers' communities of practice, pedagogy)
- What aspects of the professional learning group experience are most valuable to you, and why are these aspects of significance to you? (professional learning processes that are valued)
- How would you teach another teacher how to teach Ontario bansho? (uncover theory of practice)
- What support would you give another teacher who is trying to implement Ontario bansho? (uncover theory of practice)
- Why do you think it is important to teach and support another teacher in this/these way(s)? (make explicit the theory of practice)

- What do you perceive to be the difference between learning about mathematics-for-teaching through bansho and other professional development approaches to learning about mathematics-for-teaching? (information for emergent framework)
- Some teachers worry about not doing bansho right. How do you think about this?

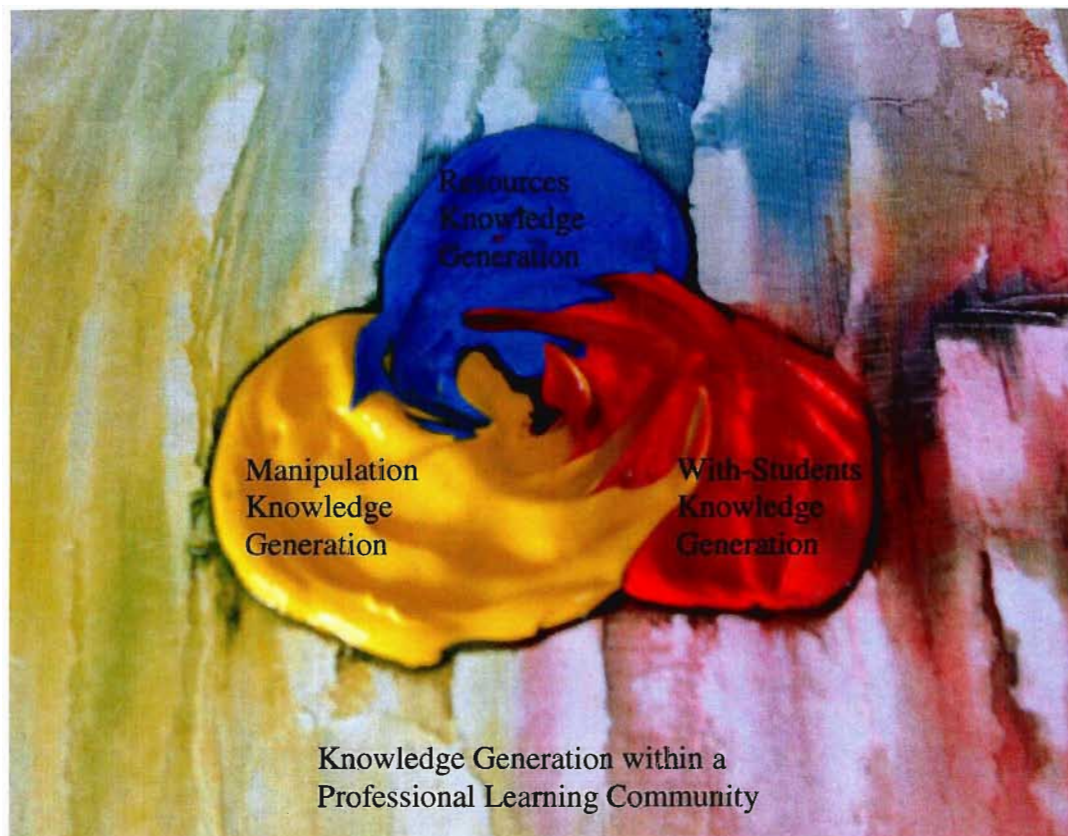
Interviewer:

- Thank you for your participation in this first interview. I will be contacting you once the interview has been transcribed to arrange for you to get a copy of the transcript. Once you receive the transcript, please confirm the accuracy of our conversation and to add or clarify any points that you wish. Once you have confirmed the accuracy of the transcript, I will begin an initial analysis of the data. I will then contact you to arrange for a second interview, again at a mutually convenient time in your classroom. Please don't hesitate to send me an e-mail if you have any questions.

Appendix B

Interview Guide for Second Interview

I am proposing the following framework of organizing ideas based on the things you said and the themes that came from the interviews I had with you and the other participants.



Interviewer:

I was looking for an image that would capture some of the more organic elements you spoke about. At first, I considered a bowl of fruit with the dish being a safe community of learners. But fruit have boundaries, and I wanted to show how themes blend and share qualities with other themes. I created a textured canvas to represent the community. I overlaid the blobs of primary colours and created the conditions so that the colours would

blend, yet leave identifiable themes. I like the way the colours blend, but in a way that would never be exactly replicable. I think this is like the knowledge we generate.

I have attached one of my working documents with the picture of the framework so that you can ‘see’ a bit of the ‘colour’ of what I was seeing from your interviews. Your input and thoughts about this framework would be greatly appreciated.

The following will be the questions I will ask when we set up our next interview time:

- All of the participants said that being a part of the groups made them feel safe and that this safety figured prominently in their risk-taking and willingness to be vulnerable in their practice. What kinds of things do you think about when you feel safe? What kinds of things do you think about when you don’t feel safe? What is the difference for you?
- Most participants said that they questioned students differently. What kinds of thinking processes do you go through as you consider how you pose questions?
- What role does manipulating things (number lines, manipulatives, arrays, etc.) play in your own math learning? What do you think about as you are manipulating these things? What role does manipulating things (number lines, manipulatives, arrays, etc.) play in the learning of your students? In what ways to these manipulations contribute to making connections among mathematical ideas?
- What role does manipulating ideas about mathematics teaching and learning play in your own math learning? What do you think about as you are considering

ideas about mathematics teaching and learning? In what ways does this manipulating of ideas contribute to making connections among mathematical ideas?

- The participants spoke about valuing the opportunity to see a knowledgeable other do a bansho. What do you want to get from observing another person do the bansho? What are you looking for when you're observing? What do you get when you observe? What are you thinking about during observation? What do you think about after? What is satisfying to you about this observing and thinking process?
- Some participants spoke about valuing reading research and recommended professional resources. What do you want to get from professional reading? What are you looking for when you do professional reading? What **do** you get when you do professional reading? What are you thinking about during professional reading? What do you think about after? What is satisfying to you about this reading and thinking process?
- How does experiencing personal satisfaction with something you're learning cause you to think differently than when you are bored or irritated by something you're supposed to be learning? In what ways is your thinking different? What kinds of thinking do you do that is different?
- Do you see the framework applying to other areas of learning, like literacy? What might be the physical things that would be manipulated? What ideas?

- Do you feel legitimized in your school?
- What personal satisfaction do you have as a result of the legitimization in your school?
- What personal struggles do you have as a result of the lack of legitimization in your school?
- What is the biggest barrier to legitimizing our work in your school? Area? Board?
- Is this the elephant in the room?

Appendix C

Research Ethics Board Clearance Letter



**Brock
University**

Office of Research Services
Research Ethics Office
St. Catharines, Ontario, Canada L2S 5A1
T: 905.688-5550 Ext. 3035/4876 F: 905.688-0748

www.brocku.ca

DATE: December 5, 2008

FROM: Michelle McGinn, Chair
Research Ethics Board (REB) *Michelle K. McGinn*

TO: Coral Mitchell, Education
Glynis Fleming

FILE: 08-154 MITCHELL/FLEMING
Masters Thesis/Project

TITLE: Ontario Bansho: Informing Teachers' Professional Learning Process Concerning Mathematics-for-Teaching

The Brock University Research Ethics Board has reviewed the above research proposal.

DECISION: ACCEPTED WITH NOTES

Please note:

- Novice users may be below level in terms of their instructional effect at facilitating this strategy. You might raise this issue so that they will not feel vulnerable. You might wish to consider providing novice users with feedback that also enhances their professional learning and development such that they might be more accomplished in these strategies.
- Please submit a copy of the DSBN exemption once it has been obtained.

This project has received ethics clearance for the period of **December 5, 2008 to March 31, 2010** subject to full REB ratification at the Research Ethics Board's next scheduled meeting. The clearance period may be extended upon request. ***The study may now proceed.***

Please note that the Research Ethics Board (REB) requires that you adhere to the protocol as last reviewed and cleared by the REB. During the course of research no deviations from, or changes to, the protocol, recruitment, or consent form may be initiated without prior written clearance from the REB. The Board must provide clearance for any modifications before they can be implemented. If you wish to modify your research project, please refer to <http://www.brocku.ca/researchservices/forms> to complete the appropriate form Revision or Modification to an Ongoing Application.

Adverse or unexpected events must be reported to the REB as soon as possible with an indication of how these events affect, in the view of the Principal Investigator, the safety of the participants and the continuation of the protocol.

If research participants are in the care of a health facility, at a school, or other institution or community organization, it is the responsibility of the Principal Investigator to ensure that the ethical guidelines and clearance of those facilities or institutions are obtained and filed with the REB prior to the initiation of any research protocols.

The Tri-Council Policy Statement requires that ongoing research be monitored. A Final Report is required for all projects upon completion of the project. Researchers with projects lasting more than one year are required to submit a Continuing Review Report annually. The Office of Research Services will contact you when this form *Continuing Review/Final Report* is required.

Please quote your REB file number on all future correspondence.

MMWan